A.2 AASHTO Type IV, LRFD Specifications

A.2.1 INTRODUCTION

Detailed example showing sample calculations for design of typical Interior AASHTO Type–IV prestressed concrete Beam supporting single span bridge. The design is based on *AASHTO LRFD Bridge Design Specifications 3rd Edition 2004.*

A.2.2 DESIGN PARAMETERS

The bridge considered for design has a span length of 110 ft. (c/c pier distance) with no skew and a total width of 46 ft. The bridge superstructure consists of 6 AASHTO Type IV beams spaced 8 ft. center to center designed to act compositely with 8 in. thick cast in place concrete deck as shown in figure A.2.2.1. The wearing surface thickness is 1.5 in. which includes the thickness of any future wearing surface. T501 type rails are considered in the design. HL-93 is the design live load. The relative humidity of 60% is considered in the design. The bridge cross section is shown in fig A.2.2.1.

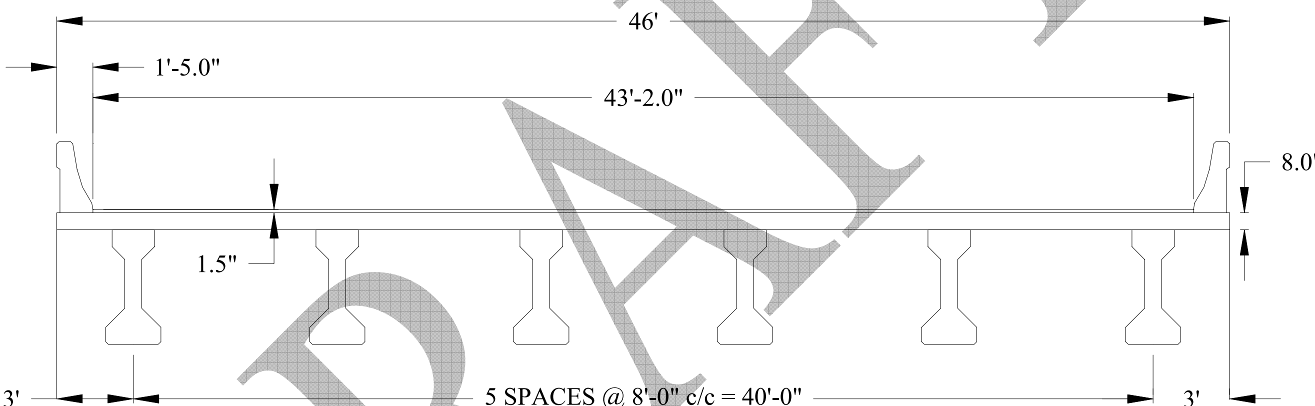

Figure A.2.2.1 Bridge Cross Section

A.2.3 MATERIAL PROPERTIES

Cast in place slab: Thickness t_s = 8.0 in.

Concrete Strength at 28-days, f_c' = 4,000 psi

Thickness of asphalt wearing surface (including any future wearing surfaces), t_w = 1.5 in.

Unit weight of concrete = 150 pcf

Precast beams: AASHTO Type- IV

Concrete Strength at release, f'_{ci} = 4000 psi (This value is taken as initial guess and will be finalized based on most optimum design)

Concrete Strength at 28 days, f'_c = 5000 psi (This value is taken as initial guess and will be finalized based on most optimum design)

Concrete unit weight = 150 pcf

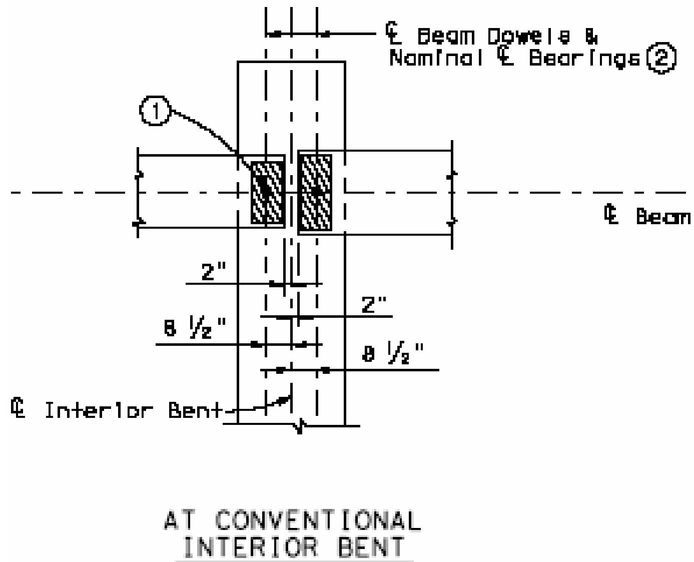

Fig A.2.3.1 Beam end Details
(Adapted from TxDOT Standard drawing ibebste1)

Span Length (c/c Piers) = 110'-0"

From fig. A.2.3.1

Overall beam length = 110' – 2(2") = 109'-8"

Design Span = 110' – 2(8.5") = 108'-7" = 108.583'(c/c of bearing)

Pretensioning Strands: ½ in. diameter, seven wire low relaxation

Area of one strand = 0.153 in.^{2}

Ultimate Stress, f_{pu} = 270,000 psi

Yield Strength, $f_{py} = 0.9f_{pu}$ = 243,000 psi [LRFD Table 5.4.4.1-1]

Stress limits for prestressing strands: [LRFD Table 5.9.3-1]

before transfer, $f_{pi} \leq 0.75\ f_{pu}$ = 202,500 psi

at service limit state(after all losses) $f_{pe} \leq 0.80\ f_{py}$ = 194,400 psi

Modulus of Elasticity, E_p = 28,500 ksi [LRFD Art. 5.4.4.2]

Non Prestressed Reinforcement:

Yield Strength, f_y = 60,000 psi

Modulus of Elasticity, E_s = 29,000 ksi [LRFD Art. 5.4.3.2]

Unit weight of asphalt wearing surface = 140 pcf

T501 type barrier weight = 326 plf /side

A.2.4 CROSS-SECTION PROPERTIES FOR A TYPICAL INTERIOR BEAM

A.2.4.1 Non-Composite Section

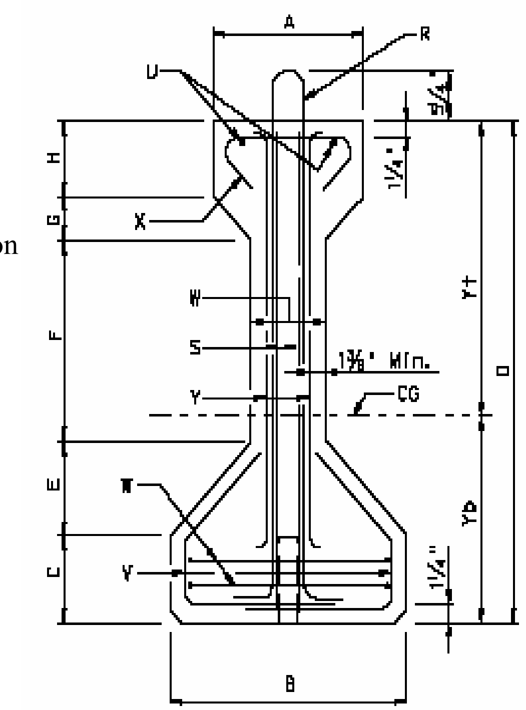

Figure A.2.4.1 Section Geometry of AASHTO Type – IV Beams (Adapted from TxDOT 2001)

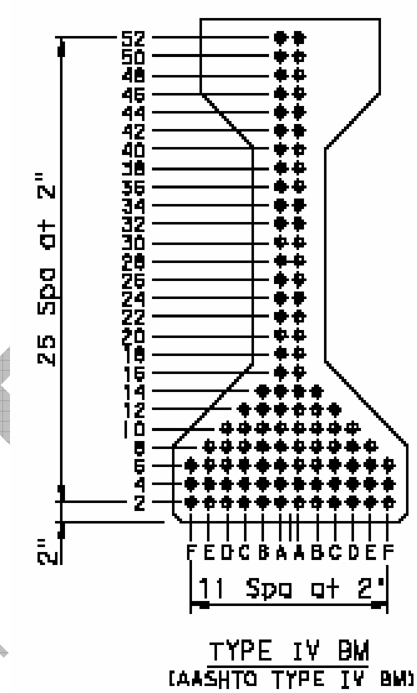

Figure A.2.4.2 Strand Pattern for AASHTO Type – IV Beams (Adapted from TxDOT 2001)

Table A.2.4.1 Section Properties of AASHTO Type IV beam (notations as used in Figure A.2.4.1, Adapted from TxDOT Bridge Design Manual)

A	B	C	D	E	F	G	H	W	y_t	y_b	Area	I	Wt/lf
in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.^{2}	in.^{4}	lbs
20.00	26.00	8.00	54.00	9.00	23.00	6.00	8.00	8.00	29.25	24.75	788.40	260,403.00	821.00

where

I = moment of inertia about the centroid of the non-composite Precast beam

y_b = distance from centroid to the extreme bottom fiber of the non-composite precast beam

y_t = distance from centroid to the extreme top fiber of the non-composite precast beam

S_b = section modulus for the extreme bottom fiber of the non-composite precast beam = I/y_b = 260403.00/24.75 = 10521.33 in.^{3}

S_t = section modulus for the extreme top fiber of the non-composite precast beam = I/y_t = 260403.00/29.25 = 8902.67 in.^{3}

A.2.4.2
Composite Section

A.2.4.2.1
Effective Flange Width

The effective flange width is lesser of: [LRFD Art. 4.6.2.6.1]

1/4 span length: $\dfrac{108.583(12\text{ in./ft})}{4} = 325.75$ in.

Distance center to center of beams: 8(12 in./ft) = 96.00 in. (controls)

12(Effective slab thickness) + greater of web thickness or ½ beam top flange width: 12(8.0) + 1/2(20.0) = 106.00 in.

Effective flange width = 96.00 in.

A.2.4.2.2
Modular Ratio between Slab and Beam Material

Following the TxDOT Design manual recommendation (Pg. #7-85) the modular ratio between slab and beam materials is taken as 1

$$n = \left(\frac{E_c \text{ for slab}}{E_c \text{ for beam}}\right) = 1$$

A.2.4.2.3
Transformed Section Properties

Transformed flange width = n (effective flange width) = 1(96) = 96.0 in.

Transformed Flange Area = n (effective flange width) (t_s) = 1(96) (8) = 768.0 in.^{2}

[LRFD]

Table A.2.4.2 Properties of Composite Section

	Transformed Area in.^{2}	y_b in.	A y_b in.	$A(y_{bc} - y_b)^2$	I in^4	$I+A(y_{bc} - y_b)^2$ in^4
Beam	788.40	24.75	19512.90	212231.53	260403.00	472634.53
Slab	768.00	58.00	44544.00	217868.93	4096.00	221964.93
Σ	1556.40		64056.90			694599.46

A_c = total area of composite section = 1556.4 in.^{2}

h_c = total height of composite section = 62.0 in.

I_c = moment of inertia of composite section = 694599.5 in^4

y_{bc} = distance from the centroid of the composite section to extreme bottom fiber of the precast beam = 64056.9/1556.4 = 41.157 in.

y_{tg} = distance from the centroid of the composite section to extreme top fiber of the precast beam = 54 - 41.157 = 12.843 in.

y_{tc} = distance from the centroid of the composite section to extreme top fiber of the slab = 62 - 41.157 = 20.843 in.

S_{bc} = composite section modulus for extreme bottom fiber of the precast beam = I_c/y_{bc} = 694599.50/41.157 = 16876.83 in.^{3}

S_{tg} = composite section modulus for top fiber of the precast beam

= I_c/y_{tg} = 694599.50/12.843 = 54083.90 in.^{3}

S_{tc} = composite section modulus for top fiber of the slab

$$= \left(\frac{1}{n}\right) I_c/y_{tc} = 1(694599.50/20.843) = 33325.31 \text{ in.}^3$$

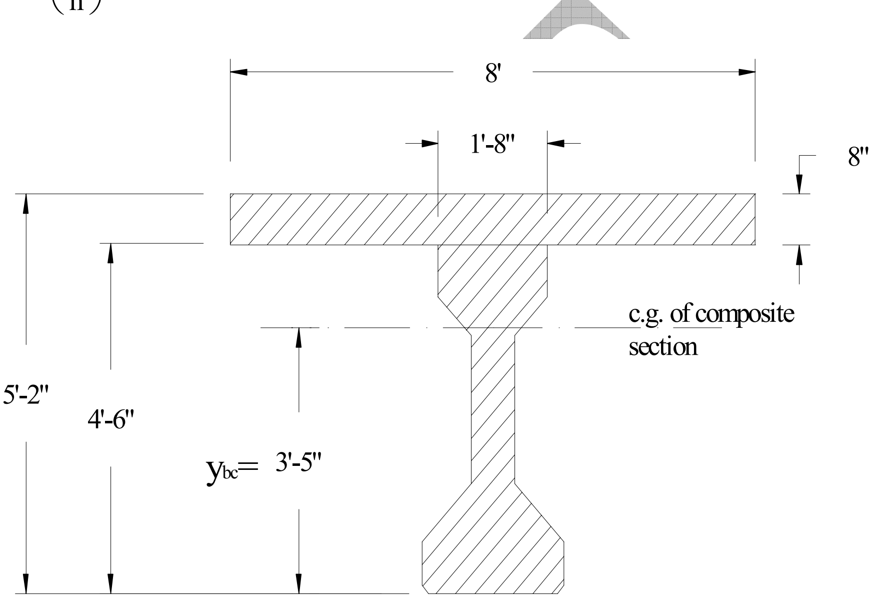

Figure A.2.4.3 Composite Section

A.2.5 SHEAR FORCES AND BENDING MOMENTS

The self weight of the beam and the weight of slab act on the non-composite simple span structure, while the weight of barriers, future wearing surface, and live load plus impact act on the composite simple span structure

A.2.5.1 Shear Forces and Bending Moments due to Dead Loads

A.2.5.1.1 Dead Loads

[LRFD Art. 3.3.2]

DC = Dead load of structural components and non-structural attachments

Dead loads acting on the non-composite structure:

Self Weight of the beam = 0.821 kip/ft. (TxDOT Bridge Design Manual)

Weight of cast in place deck on each interior beam = $(0.150 \text{ pcf})\left(\frac{8''}{12 \text{ in/ft}}\right)(8')$

= 0.800 kip/ft.

Total Dead Load = 0.821 + 0.800 = 1.621 kips/ft.

A.2.5.1.2 Super Imposed Dead Load

Dead loads placed on the composite structure:

The permanent loads on the bridge including loads from railing and wearing surface can be distributed uniformly among all beams if the following conditions are met: [LRFD Art. 4.6.2.2.1]

Width of deck is constant (O.K.)

Number of beams, N_b, is not less than four ($N_b = 6$) (O.K.)

Beams are parallel and have approximately the same stiffness (O.K.)

The roadway part of the overhang, $d_e \leq 3.0$ ft.

d_e = 3.0 – (width of barrier at road surface) = 3 .0 – 1.417 = 1.583 ft. (O.K.)

Curvature in plan is less than 4^0 (curvature = 0.0) (O.K.)

Cross section of the bridge is consistent with one of the cross sections given in LRFD Table 4.6.2.2.1-1 (O.K.)

Since all the above criteria are satisfied, the barrier and wearing surface loads are equally distributed among the 6 beams

Weight of T501 Rails or Barriers on each beam = $2\left(\dfrac{326 \text{ plf}/1000}{6 \text{ beams}}\right)$

= 0.110 kips/ft./beam

Weight of 1.5" Wearing surface = $(0.140 \text{ kcf})\left(\dfrac{1.5"}{12 \text{ in/ft}}\right)$ = 0.0175 kips/ft.

Weight of wearing surface on each beam = $\dfrac{(0.0175 \text{ ksf})(43.167 \text{ ft.})}{6 \text{ beams}}$

= 0.126 Kips/ft./beam

Total Super Imposed Dead Load = 0.110 + 0.126 = 0.236 kip/ft./beam

A.2.5.1.3 Unfactored Shear Forces and Bending Moments

Shear forces and bending moments for the beam due to dead loads, superimposed dead loads at every tenth of the design span and at critical sections (hold down point or harp point) are shown in this section. The bending moment (M) and shear force (V) due to dead loads and super imposed dead loads at any section at a distance x are calculated using the following formulae.

$$M = 0.5wx\,(L - x)$$

$$V = w(0.5L - x)$$

As per the recommendations of TxDOT Bridge Design Manual Chap. 7, Sec 21

Distance of hold down point from centerline of bearing = $\dfrac{108.583}{2} - \dfrac{108.583}{20}$

HD = 48.862 ft.

Table A.2.5.1. Shear forces and Bending moments due to Dead loads

Distance	Section	Dead Load				Super Imposed Dead Loads						Total Dead Load	
		Beam Weight		Slab Weight		Barrier		Wearing Surface		Total			
x	x/L	Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment
ft.		Kips	k-ft	kips	k-ft	kips	k-ft	kips	k-ft.	kips	k-ft.	Kips	K-ft.
0.000	0.0	44.57	0.00	43.43	0.00	5.97	0.00	6.84	0.00	12.81	0.00	100.81	0.00
5.520	0.051	40.04	233.54	39.02	227.56	5.36	31.29	6.15	35.84	11.51	67.13	85.21	496.94
6.190	0.057	39.49	260.18	38.48	253.53	5.29	34.86	6.06	39.93	11.35	74.79	89.32	588.50
10.858	0.1	35.66	435.58	34.75	424.44	4.78	58.36	5.47	66.85	10.25	125.21	80.66	985.23
21.717	0.2	26.74	774.40	26.06	754.59	3.58	103.76	4.10	118.85	7.69	222.60	60.49	1751.59
32.575	0.3	17.83	1016.38	17.37	990.38	2.39	136.18	2.74	155.99	5.13	292.16	40.33	2298.92
43.433	0.4	8.91	1161.58	8.69	1131.86	1.19	155.63	1.37	178.27	2.56	333.90	20.16	2627.34
48.862	0.45	4.46	1197.87	4.34	1167.24	0.60	160.49	0.68	183.84	1.28	344.33	10.08	2709.44
54.292	0.5	0.00	1209.98	0.00	1179.03	0.00	162.11	0.00	185.70	0.00	347.81	0.00	2736.82

A.2.5.2 Shear Forces and Bending Moments due to Live Load

A.2.5.2.1 Live Load

[LRFD Art. 3.6.1.2.1]

Design live load is HL-93 which consists of a combination of:

1. Design truck or design tandem with dynamic allowance [LRFD Art. 3.6.1.2.2]

The design truck is the same as HS20 design truck specified by the Standard Specifications, [STD Art. 3.6.1.2.2]. The design tandem consists of a pair of 25.0-kip axles spaced at 4.0 ft. apart. [LRFD Art. 3.6.1.2.3]

2. Design lane load of 0.64 kip/ft. without dynamic allowance

[LRFD Art. 3.6.1.2.4]

A.2.5.2.2 Live Load Distribution Factor for a Typical Interior Beam

The bending moments and shear forces due to live load are determined using simplified distribution factor formulas, [LRFD Art. 4.6.2.2]. To use the simplified live load distribution factors the following conditions must be met

[LRFD Art. 4.6.2.2.1]

Width of deck is constant (O.K.)

Number of beams, N_b, is not less than four ($N_b = 6$) (O.K.)

Beams are parallel and of the same stiffness (O.K.)

The roadway part of the overhang, $d_e \leq 3.0$ ft.

d_e = 3.0 – (width of barrier at road surface) = 3 .0 – 1.417 = 1.583 ft. (O.K.)

Curvature in plan is less than 4^0 (curvature = 0.0) (O.K.)

For precast concrete I-beams with cast-in-place concrete deck, the bridge type is (k) [LRFD Table 4.6.2.2.1-1]

The number of design lanes is computed as:

Number of design lanes = the integer part of the ratio of (w/12), where (w) is the clear roadway width, in ft., between the curbs [LRFD Art. 3.6.1.1.1]

w = 43.167

Number of design lanes = integer part of (43.167/12) = 3 lanes

A.2.5.2.2.1 Distribution factor for Bending Moment

For all limit states except fatigue limit state:

For two or more lanes loaded:

$$\text{DFM} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1} \qquad \text{[LRFD Table 4.6.2.2.2b-1]}$$

Provided that: $3.5 \le S \le 16$; $S = 8.0$ ft (O.K.)

$4.5 \le t_s \le 12$; $t_s = 8.0$ in (O.K.)

$20 \le L \le 240$; $L = 108.583$ ft. (O.K.)

$N_b \ge 4$; $N_b = 6$ (O.K.)

$10{,}000 \le K_g \le 7{,}000{,}000$ (O.K., as shown below)

where

DFM = distribution factor for moment for interior beam

S = spacing of beams = 8.0 ft

L = beam span = 108.583 ft

t_s = depth of concrete deck = 8.0 in.

K_g = longitudinal stiffness parameter, in.^{4}

$K_g = n\,(I + Ae_g^2)$ [LRFD Art. 3.6.1.1.1]

where

n = modular ratio between beam and slab materials

$$= \frac{E_c(\text{beam})}{E_c(\text{deck})} = 1$$

The modular ratio is assumed to be 1 and needs to be updated once the optimum f_c' value is established, and the distribution factor based on new modular ratio will be compared to the present distribution factor and updated if needed

A = cross-sectional area of the beam (non-composite section)

A = 788.4 in.^{2}

I = moment of inertia of beam (non-composite section) = 260,403.0 in.^{4}

e_g = distance between centers of gravity of the beam and slab, in.

$= (t_s/2 + y_t) = (8/2 + 29.25) = 33.25$ in.

Therefore

$$K_g = 1[260403 + 788.4(33.25)^2] = 1{,}132{,}028.48 \text{ in.}^4$$

$$DFM = 0.075 + \left(\frac{8}{9.5}\right)^{0.6}\left(\frac{8}{108.583}\right)^{0.2}\left(\frac{1132028.48}{12.0(108.583)(8)^3}\right)^{0.1}$$

$$= 0.075 + (0.902)(0.593)(1.054) = 0.639 \text{ lanes/beam}$$

For one design lane loaded:

$$DFM = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$

$$= 0.06 + \left(\frac{8}{14}\right)^{0.4}\left(\frac{8}{108.583}\right)^{0.3}\left(\frac{1132028.48}{12.0(108.583)(8)^3}\right)^{0.1}$$

$$= 0.06 + (0.8)(0.457)(1.054) = 0.445 \text{ lanes/beam}$$

Thus, the case of two or more lanes loaded controls

DFM = 0.639 lanes/beam

For fatigue limit state:

LRFD Specifications, Art. 3.4.1, states that for fatigue limit state, a single design truck should be used. However, live load distribution factors given in LRFD Article 4.6.2.2 take into consideration the multiple presence factor, m. LRFD Article 3.6.1.1.2 states that the multiple presence factor, m, for one design lane loaded is 1.2. Therefore, the distribution factor for one design lane loaded with the multiple presence factor removed should be used. The distribution factor for the fatigue limit state is: 0.445/1.2 = 0.371 lanes/beam.

A.2.5.2.2.2 Distribution factor for Shear Force

For two or more lanes loaded:

$$DFV = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^2 \qquad \text{[LRFD Table 4.6.2.2.3a-1]}$$

Provided that: $3.5 \le S \le 16$; S = 8.0 ft (O.K.)

$4.5 \le t_s \le 12$; t_s = 8.0 in (O.K.)

$20 \le L \le 240$; L = 108.583 ft. (O.K.)

$N_b \ge 4$; $N_b = 6$ (O.K.)

where

DFV = Distribution factor for shear for interior beam

S = Beam spacing = 8 ft.

Therefore the distribution factor for shear force is:

$$DFV = 0.2 + \left(\frac{8}{12}\right) - \left(\frac{8}{35}\right)^2 = 0.814 \text{ lanes/beam}$$

For one design lane loaded:

$$DFV = 0.36 + \left(\frac{S}{25.0}\right) = 0.36 + \left(\frac{8}{25.0}\right)$$ = 0.68 lanes/beam [LRFD Table 4.6.2.2.3a-1]

Thus, the case of two or more lanes loaded controls

DFV = 0.814 lanes/beam

A.2.5.2.3 Dynamic Allowance

IM = 33% [LRFD Table 3.6.2.1-1]

where

IM = dynamic load allowance, applied to truck load only

A.2.5.2.4 Unfactored Shear Forces and Bending Moments

A.2.5.2.4.1 Due to Truck load V_{LT} and M_{LT}

For all limit states except for fatigue limit state:

Shear force and bending moment due to truck load on a per-lane-basis are calculated at tenth-points of the span using the following equations

For x/L = 0 – 0.333

Maximum bending moment due to truck load, $M = \dfrac{72(x)[(L - x) - 9.33]}{L}$

For x/L = 0.333 – 0.5

Maximum bending moment due to truck load, $M = \dfrac{72(x)[(L - x) - 4.67]}{L} - 112$

Unfactored Bending Moment due to truck load plus impact

M_{LT} = (bending moment per lane) (DFM) (1+IM)

= (bending moment per lane)(0.639)(1+0.33)

= (bending moment per lane)(0.85)

For x/L = 0 – 0.5

Maximum Shear Force due to truck load, $V = \dfrac{72[(L - x) - 9.33]}{L}$

Unfactored Shear Force due to truck load plus impact

V_{LT} = (shear force per lane)(DFV) (1+IM)

= (shear force per lane)(0.814)(1 + 0.33)

= (shear force per lane)(1.083)

M_{LT} and V_{LT} values at every tenth of span are shown in Table A.2.5.2

For fatigue limit state:
Art. 3.6.1.4.1 in the LRFD specifications states that the fatigue load is a single design truck which has the same axle weight used in all other limit states but with a constant spacing of 30.0 ft. between the 32.0 kip axles. Bending moment envelope on a per-lane-basis is calculated using the following equation

For x/L = 0 – 0.241

Maximum bending moment due to fatigue truck load, $M = \dfrac{72(x)[(L - x) - 18.22]}{L}$

For x/L = 0.241 – 0.5

$$M = \frac{72(x)[(L - x) - 11.78]}{L} - 112$$

Unfactored Bending Moment due to fatigue truck load plus impact

M_f = (bending moment per lane) (DFM) (1+IM)

= (bending moment per lane)(0.371)(1+0.15)

= (bending moment per lane)(0.427)

M_f values at every tenth of span are shown in Table A.2.5.2

A.2.5.2.4.1 Due to Design Lane Load V_{LL} and M_{LL}

The maximum bending moments and shear forces due to uniformly distributed lane load of 0.64 kip/ft. are calculated using the following formulae

Maximum Bending moment, M_x = 0.5(0.64)(x)(L-x)

Unfactored Bending Moment due to lane load

M_{LL} = (bending moment per lane) (DFM)

= (bending moment per lane)(0.639)

Maximum Shear Force, $V_x = \dfrac{0.32(L - x)^2}{L}$ for $x \leq 0.5L$

Unfactored Shear Force due to lane load

V_{LL} = (shear force per lane)(DFV)

= (shear force per lane)(0.814)

where

x = distance from the support to the section at which bending moment or shear force is calculated

L = span length = 108.583 ft.

Table A.2.5.2. Shear forces and Bending moments due to Live loads

Distance x	Section x/L	Truck Loading				Lane loading		Fatigue Truck loading	
		Truck Load		Truck load + Impact				Truck load	Truck load + impact
		Shear	Moment	Shear	Moment	Shear	Moment	Moment	Moment
				V_{LT}	M_{LT}	V_{LL}	M_{LL}		M_f
ft.		Kips	K-ft.	Kips	K-ft.	Kips	K-ft.	K-ft.	K-ft.
0	0	65.81	0.00	71.25	0.00	28.28	0.00	0.00	0.00
5.520	0.051	62.15	343.09	67.28	291.58	25.48	116.33	310.55	132.50
6.190	0.057	61.71	381.98	66.81	324.63	25.15	129.60	345.49	147.40
10.858	0.1	58.61	636.43	63.45	540.88	22.91	216.97	572.42	244.22
21.717	0.2	51.41	1116.54	55.66	948.91	18.10	385.75	988.52	421.75
32.575	0.3	44.21	1440.25	47.86	1224.03	13.86	506.28	1275.33	544.12
43.433	0.4	37.01	1629.82	40.07	1385.14	10.18	578.61	1425.05	608.00
48.862	0.45	33.41	1671.64	36.17	1420.68	8.56	596.69	1441.28	614.92
54.292	0.5	29.81	1674.37	32.27	1423.00	7.07	602.72	1418.41	605.16

A.2.5.3
Load Combinations

Total factored load shall be taken as:

$Q = \eta\sum\gamma_i q_i$ [LRFD Eq. 3.4.1-1]

where

η = a factor relating to ductility, redundancy and operational importance

(η = 1 in present case) [LRFD Art. 1.3.2]

γ_i = load factors [LRFD Table 3.4.1-1]

q_i = specified loads

Investigating different limit states given in LRFD Art. 3.4.1, the following limit states are applicable in present case:

Service I: check compressive stresses in prestressed concrete components:

Q = 1.00(DC + DW) + 1.00(LL+IM) [LRFD Table 3.4.1-1]

This load combination is the general combination for service limit state stress checks and applies to all conditions other than Service III.

Service III: check tensile stresses in prestressed concrete components:

Q = 1.00(DC + DW) + 0.80(LL+IM) [LRFD Table 3.4.1-1]

This load combination is a special combination for service limit state stress checks that applies only to tension in prestressed concrete structures to control cracks.

Strength I: check ultimate strength: [LRFD Table 3.4.1-1 and 2]

Maximum Q = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)

Minimum Q = 0.90(DC) + 0.65(DW) + 1.75(LL + IM)

This load combination is the general load combination for strength limit state design.

For simple span bridges, the maximum load factors produce maximum effects. However, minimum load factors are used for dead load (DC), and wearing surface load (DW) when dead load and wearing surface stresses are opposite to those of live load.

Fatigue: check stress range in strands: [LRFD Table 3.4.1-1]

Q = 0.75(LL + IM)

This load combination is a special load combination to check the tensile stress range in the strands due to live load and dynamic load allowance

where

DC = self weight of beam

DW = wearing surface load

LL = live load

IM = Dynamic allowance

A.2.6 ESTIMATION OF REQUIRED PRESTRESS

The required number of strands is usually governed by concrete tensile stresses at the bottom fiber for load combination at Service III at the section of maximum moment or at the harp points. For estimating the number of strands, only the stresses at midspans are considered.

A.2.6.1 Service load Stresses at Midspan

Bottom tensile stresses at midspan due to applied dead and live loads using load combination Service III is:

$$f_b = \frac{M_g + M_S}{S_b} + \frac{M_{SDL} + (0.8)(M_{LT} + M_{LL})}{S_{bc}}$$

where

f_b = concrete stress at the bottom fiber of the beam

M_g = Unfactored bending moment due to beam self-weight

M_S = Unfactored bending moment due to slab weight

M_{SDL} = Unfactored bending moment due to super imposed dead load

M_{LT} = Bending moment due to truck load plus impact

M_{LL} = Bending moment due to lane load

Substituting the bending moments and section modulus values, bottom fiber stresses at mid span is:

$$f_{bc} = \frac{(1209.98 + 1179.03)(12)}{10521.33} + \frac{(347.81)(12) + (0.8)(1423.00 + 602.72)(12)}{16876.83}$$

$$= 2.725 + 1.400 = 4.125 \text{ ksi}$$

A.2.6.2 Allowable Stress Limit

At service load conditions, allowable tensile stress is

$F_b = 0.19\sqrt{f_c'}$ [LRFD Art. 5.9.4.2b]

where f_c' = beam concrete strength at service, ksi

$F_b = 0.19\sqrt{5} = -0.425$ ksi

A.2.6.3 Required Number of Strands

Required precompressive stress in the bottom fiber after losses:

Bottom tensile stress – allowable tensile stress at final = $f_b - F_b$

$f_{pb} = 4.125 - 0.425 = 3.700$ ksi

Assuming the distance from the center of gravity of strands to the bottom fiber of the beam is equal to $y_{bs} = 2$ in.

Strand Eccentricity at midspan:

$$e_c = y_b - y_{bs} = 24.75 - 2 = 22.75 \text{ in.}$$

Bottom fiber stress due to prestress after losses:

$$f_b = \frac{P_{pe}}{A} + \frac{P_{pe}\, e_c}{S_b}$$

where P_{pe} = effective prestressing force after all losses

$$3.700 = \frac{P_{pe}}{788.4} + \frac{22.75\, P_{pe}}{10521.33}$$

Solving for P_{pe} we get,

$P_{pe} = 1078.5$ Kips

Assuming final losses = 20% of f_{pi}

Assumed final losses = 0.2(202.5) = 40.5 ksi

The prestress force per strand after losses

= (cross sectional area of one strand) [f_{pi} – losses]

= 0.153(202.5 – 40.5) = 24.78 Kips

Number of Strands Required = 1078.5/24.78 = 43.52

Try 44 – ½ in. diameter, 270 ksi strands as an initial trial

Strand eccentricity at midspan after strand arrangement

$$e_c = 24.75 - \frac{12(2+4+6) + 8(8)}{44} = 20.02 \text{ in.}$$

P_{pe} = 44(24.78) = 1090.32 Kips

$$f_b = \frac{1090.32}{788.4} + \frac{20.02(1090.32)}{10521.33}$$

$= 1.383 + 2.075 = 3.458 \text{ ksi} < f_{pb} \text{ reqd.} = 3.700 \text{ ksi}$ (N.G.)

Therefore try 46 strands

Effective strand eccentricity at midspan after strand arrangement

$$e_c = 24.75 - \frac{12(2+4+6) + 10(8)}{46} = 19.88 \text{ in.}$$

P_{pe} = 46(24.78) = 1139.88 Kips

$$f_b = \frac{1139.88}{788.40} + \frac{19.88(1139.88)}{10521.33}$$

$= 1.446 + 2.154 = 3.600 \text{ ksi} < f_{pb} \text{ reqd.} = 3.700 \text{ ksi}$ (N.G.)

Therefore try 48 strands

Effective strand eccentricity at midspan after strand arrangement

$$e_c = 24.75 - \frac{12(2+4+6) + 10(8)+2(10)}{48} = 19.67 \text{ in.}$$

P_{pe} = 48(24.78) = 1189.44 Kips

$$f_b = \frac{1189.44}{788.4} + \frac{19.67(1189.44)}{10521.33}$$

$= 1.509 + 2.223 = 3.732 \text{ ksi} > f_{pb} \text{ reqd.} = 3.700 \text{ ksi}$ (O.K.)

Therefore use 48 strands

Number of Strands	Distance from bottom (in.)
2	10
10	8
12	6
12	4
12	2

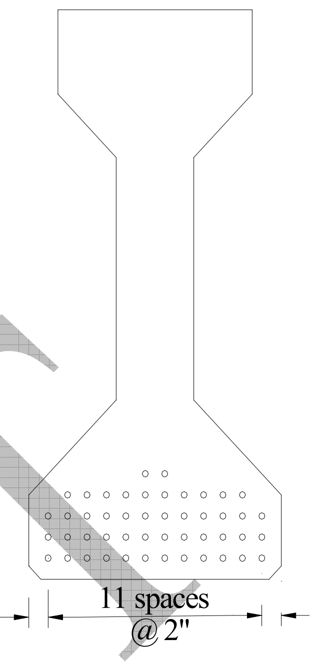

Fig. A.2.6.1 Initial Strand Pattern

A.2.7
PRESTRESS LOSSES

Total prestress loss:

$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}$ [LRFD Eq. 5.9.5.1-1]

where

Δf_{pES} = loss of prestress due to elastic shortening

Δf_{pSR} = loss of prestress due to concrete shrinkage

Δf_{pCR} = loss of prestress due to creep of concrete

Δf_{pR2} = loss of prestress due to relaxation of steel after transfer

Number of strands = 48

A number of iterations will be performed to arrive at the optimum f_c' and f'_{ci}

A.2.7.1
Iteration 1
A.2.7.1.1
Elastic Shortening

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Art. 5.9.5.2.3a]

where

f_{cgp} = sum of concrete stresses at the center of gravity of the prestressing steel due to prestressing force at transfer and self weight of the member at sections of maximum moment

LRFD Art. 5.9.5.2.3a states that f_{cgp} can be calculated on the basis of prestressing steel stress assumed to be $0.7f_{pu}$ for low-relaxation strands. However, common practice assumes the initial losses as a percentage of the initial prestressing stress before release, f_{pi}. In both procedures initial losses assumed should be checked and if different from the assumed value a second iteration should be carried out. In this example, 8% f_{pi} initial loss is used

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

where

P_i = pretension force after allowing for the initial losses, assuming 8% initial losses = (number of strands)(area of each strand)[0.92(f_{pi})]
= 48(0.153)(0.92)(202.5) = 1368.19 Kips

M_g = Unfactored bending moment due to beam self weight = 1209.98 K-ft.

e_c = eccentricity of the strand at the midspan = 19.67 in.

$$f_{cgp} = \frac{1368.19}{788.4} + \frac{1368.19(19.67)^2}{260403} - \frac{1209.98(12)(19.67)}{260403}$$

= 1.735 + 2.033 – 1.097 = 2.671 ksi

E_{ci} = modulus of elasticity of beam at release = $(w_c)^{1.5}(33)\sqrt{f'_{ci}}$

where [LRFD Eq. 5.4.2.4-1]

w_c = weight of concrete = 150 pcf

Initial assumed value of f'_{ci} = 4000 psi

$$E_{ci} = (150)^{1.5}(33)\sqrt{4000}\ \frac{1}{1000} = 3834 \text{ ksi}$$

E_p = modulus of elasticity of prestressing reinforcement = 28500 ksi

Therefore loss due to elastic shortening:

$$\Delta f_{pES} = \frac{28500}{3834}\ (2.671) = 19.855 \text{ ksi}$$

A.2.7.1.2 Shrinkage

Δf_{pSR} = 17 – 0.15 H [LRFD Eq. 5.9.5.4.2-1]

where H is relative humidity = 60%

Δf_{pSR} = [17 – 0.15(60)] = 8.0 ksi

A.2.7.1.3 Creep of Concrete

$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp}$ [LRFD Eq. 5.9.5.4.3-1]

where

Δf_{cdp} = change of stresses at the center of gravity of the prestressing steel due to permanent loads except the dead load present at the time the prestress force is applied calculated at the same section as f_{cgp}

$$\Delta f_{cdp} = \frac{M_S e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

where

M_S = slab moment = 1179.03 K-ft.

M_{SDL} = superimposed dead load moment = 347.81 K-ft.

y_{bc} = 41.157 in.

y_{bs} = the distance from center of gravity of the strand at midspan to the bottom of the beam = 24.75 – 19.67 = 5.08 in.

I = moment of inertia of the non-composite section =260403 in.^{4}

I_c = moment of inertia of composite section = 694599.5 in.^{4}

$$\Delta f_{cdp} = \frac{1179.03(12)(19.67)}{260403} + \frac{(347.81)(12)(41.157 - 5.08)}{694599.5}$$

Δf_{cdp} = 1.069 + 0.217 = 1.286 ksi

Δf_{pCR} = 12(2.671) – 7(1.286) = 23.05 ksi.

A.2.7.1.4 Relaxation of Prestressing Strands

A.2.7.1.4.1 Relaxation before Transfer

Initial loss due to relaxation of prestressing steel is accounted for in the beam fabrication process. Therefore, loss due to relaxation of the prestressing steel prior to transfer is not computed, i.e. $\Delta f_{pR1} = 0$. Recognizing this for pretensioned members, LRFD Article 5.9.5.1 allows the portion of the relaxation loss that occurs prior to transfer to be neglected in computing the final loss.

A.2.7.1.4.2 Relaxation after Transfer

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

Δf_{pR2} = 30% [20.0 – 0.4 Δf_{pES} – 0.2(Δf_{pSR} + Δf_{pCR})

= 0.3[20.0 – 0.4(19.855) – 0.2(8 + 23.05)] = 1.754 ksi

TxDOT Bridge Design Manual (Pg. # 7-85) recommends that 50% of the final steel relaxation loss be considered for calculation of total initial loss given as [Elastic shortening loss + 0.50(total steel relaxation loss)]

$$\text{Initial Prestress loss \%} = \frac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_{pi}}$$

$$= \frac{[19.855 + 0.5(1.754)]100}{202.5} = 10.24\% > 8\% \text{ (assumed initial prestress losses)}$$

Therefore next trial is required assuming 10.24% initial losses

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Art. 5.9.5.2.3a]

where

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

P_i = pretension force after allowing for the initial losses, assuming 10.24% initial losses = (number of strands)(area of each strand)[0.8976(f_{pi})]

= 48(0.153)(0.8976)(202.5) = 1334.87 Kips

M_g = 1209.98 K-ft.

e_c = 19.67 in.

$$f_{cgp} = \frac{1334.87}{788.4} + \frac{1334.87(19.67)^2}{260403} - \frac{1209.98(12)(19.67)}{260403}$$

= 1.693 + 1.983 – 1.097 = 2.579 ksi

E_{ci} = 3834 ksi

E_p = 28500 ksi

Therefore loss due to elastic shortening:

$$\Delta f_{pES} = \frac{28500}{3834}(2.579) = 19.17 \text{ ksi}$$

$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp}$ [LRFD Eq. 5.9.5.4.3-1]

where

f_{cgp} = 2.579 ksi

Δf_{cdp} = 1.286 ksi

Δf_{pCR} = 12(2.579) – 7(1.286) = 21.95 ksi.

Δf_{pSR} = 8.0 ksi

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

Δf_{pR2} = 30% [20.0 – 0.4 Δf_{pES} – 0.2(Δf_{pSR} + Δf_{pCR})

= 0.3[20.0 – 0.4(19.17) – 0.2(8 + 21.95)] = 1.903 ksi

$$\text{Initial Prestress loss \%} = \frac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_{pi}}$$

$$= \frac{[19.17 + 0.5(1.903)]100}{202.5} = 9.94\% < 10.24\% \text{ (assumed initial prestress losses)}$$

Therefore next trial is required assuming 9.94% initial losses

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Art. 5.9.5.2.3a]

where

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^{\,2}}{I} - \frac{(M_g\,)e_c}{I}$$

P_i = pretension force after allowing for the initial losses, assuming 9.94% initial losses = (number of strands)(area of each strand)[0.9006(f_{pi})]
= 48(0.153)(0.9006)(202.5) = 1339.34 Kips

M_g = 1209.98 K-ft.

e_c = 19.67 in.

$$f_{cgp} = \frac{1339.34}{788.4} + \frac{1339.34(19.67)^2}{260403} - \frac{1209.98(12)(19.67)}{260403}$$

= 1.699 + 1.99 – 1.097 = 2.592 ksi

E_{ci} = 3834 ksi

E_p = 28500 ksi

Therefore loss due to elastic shortening:

$$\Delta f_{pES} = \frac{28500}{3834}(2.592) = 19.27 \text{ ksi}$$

$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp}$ [LRFD Eq. 5.9.5.4.3-1]

where

f_{cgp} = 2.592 ksi

Δf_{cdp} = 1.286 ksi

Δf_{pCR} = 12(2.592) – 7(1.286) = 22.1 ksi.

Δf_{pSR} = 8.0 ksi

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

Δf_{pR2} = 30% [20.0 – 0.4 Δf_{pES} – 0.2(Δf_{pSR} + Δf_{pCR})

= 0.3[20.0 – 0.4(19.27) – 0.2(8 + 22.1)] = 1.882 ksi

$$\text{Initial Prestress loss \%} = \frac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_{pi}}$$

$$= \frac{[19.27 + 0.5(1.882)]100}{202.5} = 9.98\% \approx 9.94\% \text{ (assumed initial prestress losses)}$$

A.2.7.1.5 Total Losses at Transfer

Total Initial loss, $\Delta f_{pi} = (\Delta f_{pES} + 0.5\Delta f_{pR2}) = [19.27 + 0.5(1.882)] = 20.21$ ksi

f_{pi} = effective initial prestress = 202.5 – 20.21 = 182.29 ksi

P_i = effective pretension force after allowing for the initial losses

= 48(0.153)(182.29) = 1338.74 Kips

A.2.7.1.6 Total Losses at Service Loads

$\Delta f_{pES} = 19.270$ ksi

$\Delta f_{pSR} = 8.000$ ksi

$\Delta f_{pCR} = 22.100$ ksi

$\Delta f_{pR2} = 1.882$ ksi

Total final loss, $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}$ [LRFD Eq. 5.9.5.1-1]

$= 19.270 + 8.000 + 22.100 + 1.882 = 51.25$ ksi

or $\dfrac{51.25(100)}{202.5} = 25.31$ %

f_{pe} = effective final prestress = $f_{pi} - \Delta f_{pT}$ = 202.5 – 51.25 = 151.25 ksi

Check Prestressing stress limit at service limit state:

$f_{pe} \le 0.8 f_{py} = 0.8(243) = 194.4$ ksi > 151.25 ksi (O.K.)

Total prestressing force after all losses

$P_{pe} = 48(0.153)(151.25) = 1110.78$ Kips

A.2.7.1.7 Final Stresses at midspan

Final stress in the bottom fiber at midspan:

$$f_b = \frac{P_{pe}}{A} + \frac{P_{pe}\, e_c}{S_b}$$

$$f_b = \frac{1110.78}{788.4} + \frac{19.67(1110.78)}{10521.33}$$

$= 1.409 + 2.077 = 3.486$ ksi < f_{pb} reqd. = 3.700 ksi (N.G)

Therefore try 50 strands

$$e_c = 24.75 - \frac{12(2+4+6) + 10(8) + 4(10)}{50} = 19.47 \text{ in.}$$

$P_{pe} = 50(0.153)(151.25) = 1157.06$ Kips

$$f_b = \frac{1157.06}{788.4} + \frac{19.47(1157.06)}{10521.33}$$

$= 1.468 + 2.141 = 3.609$ ksi < f_{pb} reqd. = 3.700 ksi (N.G.)

Therefore try 52 strands

$$e_c = 24.75 - \frac{12(2+4+6) + 10(8) + 6(10)}{52} = 19.29 \text{ in.}$$

P_{pe} = 52(0.153)(151.25) = 1203.35 Kips

$$f_b = \frac{1203.35}{788.4} + \frac{19.29(1203.35)}{10521.33}$$

$= 1.526 + 2.206 = 3.732 \text{ ksi} > f_{pb}$ reqd. = 3.700 ksi (O.K.)

Therefore use 52 strands

Concrete stress at the top fiber of the beam, f_t

Under permanent loads, Service I:

$$f_t = \frac{P_{pe}}{A} - \frac{P_{pe}\, e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$= \frac{1203.35}{788.4} - \frac{19.29(1203.35)}{8902.67} + \frac{(1209.98 + 1179.03)(12)}{8902.67} + \frac{347.81(12)}{54083.9}$$

$= 1.526 - 2.607 + 3.22 + 0.077 = 2.216$ ksi

Under permanent and transient loads, Service I:

$$f_t = \frac{P_{pe}}{A} - \frac{P_{pe}\, e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}} + \frac{(M_{LT} + M_{LL})}{S_{tg}}$$

$$= 2.216 + \frac{(1423 + 602.72)12}{54083.9} = 2.216 + 0.449 = 2.665 \text{ ksi}$$

A.2.7.1.8 Initial Stresses at Hold down point

Initial Prestress

P_i = 52(0.153)(182.29) = 1450.30 Kips

Initial concrete stress at top fiber of the beam at hold down point

$$f_{ti} = \frac{P_i}{A} - \frac{P_i\, e_c}{S_t} + \frac{M_g}{S_t}$$

where

M_g = Moment due to beam self weight at hold down point = 1197.87 K-ft.

$$f_{ti} = \frac{1450.3}{788.4} - \frac{19.29(1450.3)}{8902.67} + \frac{1197.87(12)}{8902.67}$$

$= 1.839 – 3.142 + 1.615 = 0.312$ ksi

Initial concrete stress at bottom fiber of the beam at hold down point

$$f_{bi} = \frac{P_i}{A} + \frac{P_i e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1450.3}{788.4} + \frac{19.29(1450.3)}{10521.33} - \frac{1197.87(12)}{10521.33}$$

$= 1.839 + 2.659 - 1.366 = 3.132$ ksi

Compression stress limit at transfer is $0.6f'_{ci}$

Therefore, f'_{ci} reqd. $= \dfrac{3132}{0.6} = 5220$ psi

A.2.7.2
Iteration 2

Number of strands = 52

A.2.7.2.1
Elastic Shortening

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Art. 5.9.5.2.3a]

where

f_{cgp} = sum of concrete stresses at the center of gravity of the prestressing steel due to prestressing force at transfer and self weight of the member at sections of maximum moment

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

where

P_i = pretension force after allowing for the initial losses, assuming 9.98% initial losses = (number of strands)(area of each strand)[$0.9002(f_{pi})$]
= 52(0.153)(0.9002)(202.5) = 1450.3 Kips

M_g = Unfactored bending moment due to beam self weight = 1209.98 K-ft.

e_c = eccentricity of the strand at the midspan = 19.29 in.

$$f_{cgp} = \frac{1450.3}{788.4} + \frac{1450.3(19.29)^2}{260403} - \frac{1209.98(12)(19.29)}{260403}$$

$= 1.839 + 2.072 - 1.076 = 2.835$ ksi

E_{ci} = modulus of elasticity of beam at release = $(w_c)^{1.5}(33)\sqrt{f'_{ci}}$

where [LRFD Eq. 5.4.2.4-1]

w_c = weight of concrete = 150 pcf

f'_{ci} = 5220 psi

$$E_{ci} = (150)^{1.5}(33)\sqrt{5220}\ \frac{1}{1000} = 4380.12 \text{ ksi}$$

E_p = modulus of elasticity of prestressing reinforcement = 28500 ksi

Therefore loss due to elastic shortening:

$$\Delta f_{pES} = \frac{28500}{4380.12}(2.835) = 18.45 \text{ ksi}$$

A.2.7.2.2 Shrinkage

Δf_{pSR} = 17 – 0.15 H [LRFD Eq. 5.9.5.4.2-1]

where H is relative humidity = 60%

Δf_{pSR} = [17 – 0.15(60)] = 8.0 ksi

A.2.7.2.3 Creep of Concrete

$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp}$ [LRFD Eq. 5.9.5.4.3-1]

where

Δf_{cdp} = change of stresses at the center of gravity of the prestressing steel due to permanent loads except the dead load present at the time the prestress force is applied calculated at the same section as f_{cgp}

$$\Delta f_{cdp} = \frac{M_s e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

where

M_S = slab moment = 1179.03 K-ft.

M_{SDL} = superimposed dead load moment = 347.81 K-ft.

y_{bc} = 41.157 in.

y_{bs} = the distance from center of gravity of the strand at midspan to the bottom of the beam = 24.75 – 19.29 = 5.46 in.

I = moment of inertia of the non-composite section =260403 in.^{4}

I_c = moment of inertia of composite section = 694599.5 in.^{4}

$$\Delta f_{cdp} = \frac{1179.03(12)(19.29)}{260403} + \frac{(347.81)(12)(41.157 - 5.46)}{694599.5}$$

Δf_{cdp} = 1.048 + 0.214 = 1.262 ksi

Δf_{pCR} = 12(2.835) – 7(1.262) = 25.19 ksi.

A.2.7.2.4 Relaxation of Prestressing Strands

A.2.7.2.4.1 Relaxation before Transfer

Loss due to Relaxation of steel before transfer

$\Delta f_{pR1} = 0$

A.2.7.2.4.2 Relaxation after Transfer

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

$\Delta f_{pR2} = 30\% \, [20.0 - 0.4 \, \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$

$= 0.3[20.0 - 0.4(18.45) - 0.2(8 + 25.19)] = 1.795$ ksi

TxDOT Bridge Design Manual (Pg. # 7-85) recommends that 50% of the final steel relaxation loss be considered for calculation of total initial loss given as [Elastic shortening loss + 0.50(total steel relaxation loss)]

$$\text{Initial Prestress loss \%} = \frac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_{pi}}$$

$$= \frac{[18.45 + 0.5(1.795)]100}{202.5} = 9.55\% < 9.98\% \text{ (assumed initial prestress losses)}$$

Therefore next trial is required assuming 9.55% initial losses

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp} \qquad \text{[LRFD Art. 5.9.5.2.3a]}$$

where

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

P_i = pretension force after allowing for the initial losses, assuming 9.55% initial losses = (number of strands)(area of each strand)[0.9045(f_{pi})]
= 52(0.153)(0.9045)(202.5) = 1457.23 Kips

M_g = 1209.98 K-ft.

e_c = 19.29 in.

$$f_{cgp} = \frac{1457.23}{788.4} + \frac{1457.23(19.29)^2}{260403} - \frac{1209.98(12)(19.29)}{260403}$$

$= 1.848 + 2.082 - 1.076 = 2.854$ ksi

E_{ci} = 4380.12 ksi

E_p = 28500 ksi

Therefore loss due to elastic shortening:

$$\Delta f_{pES} = \frac{28500}{4380.12}(2.854) = 18.57 \text{ ksi}$$

$\Delta f_{pCR} = 12f_{cgp} - 7\Delta f_{cdp}$ [LRFD Eq. 5.9.5.4.3-1]

where

f_{cgp} = 2.854 ksi

Δf_{cdp} = 1.262 ksi

$\Delta f_{pCR} = 12(2.854) - 7(1.262) = 25.41$ ksi.

$\Delta f_{pSR} = 8.0$ ksi

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

$\Delta f_{pR2} = 30\% \, [20.0 - 0.4\,\Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$

$= 0.3[20.0 - 0.4(18.57) - 0.2(8 + 25.41)] = 1.767$ ksi

Initial Prestress loss % $= \dfrac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_{pi}}$

$= \dfrac{[18.57 + 0.5(1.767)]100}{202.5} = 9.61\% \approx 9.55\%$ (assumed initial prestress losses)

A.2.7.2.5 Total Losses at Transfer

Total Initial loss, $\Delta f_{pi} = (\Delta f_{pES} + 0.5\Delta f_{pR2}) = [18.57 + 0.5(1.767)] = 19.45$ ksi

f_{pi} = effective initial prestress = 202.5 – 19.45 = 183.05 ksi

P_i = effective pretension force after allowing for the initial losses

= 52(0.153)(183.05) = 1456.35 Kips

A.2.7.2.6 Total Losses at Service Loads

Δf_{pES} = 18.570 ksi

Δf_{pSR} = 8.000 ksi

Δf_{pCR} = 25.410 ksi

Δf_{pR2} = 1.767 ksi

Total final loss, $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}$ [LRFD Eq. 5.9.5.1-1]

$= 18.570 + 8.000 + 25.410 + 1.767 = 53.75$ ksi

or $\dfrac{53.75(100)}{202.5} = 26.54\ \%$

f_{pe} = effective final prestress = $f_{pi} - \Delta f_{pT}$ = 202.5 – 53.75 = 148.75 ksi

Check Prestressing stress limit at service limit state:

$f_{pe} \leq 0.8f_{py} = 0.8(243) = 194.4 \text{ ksi} > 148.75$ ksi (O.K.)

Total prestressing force after all losses

$P_{pe} = 52(0.153)(148.75) = 1183.46$ Kips

A.2.7.2.7
Final Stresses at midspan

Concrete stress at the top fiber of the beam, f_t

Effective prestress and permanent loads, Service I:

$$f_t = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$= \frac{1183.46}{788.4} - \frac{19.29(1183.46)}{8902.67} + \frac{(1209.98 + 1179.03)(12)}{8902.67} + \frac{347.81(12)}{54083.9}$$

= 1.501 - 2.564 + 3.220 + 0.077 = 2.234 ksi

Allowable compression stress limit for this load combination = 0.45f'$_c$

f'$_c$ reqd. = 2234/0.45 = 4964.44 psi

1/2(Effective prestress + permanent loads) + transient loads, Service I:

$$f_t = 0.5\left(\frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}\right) + \frac{(M_{LT} + M_{LL})}{S_{tg}}$$

$$= 0.5(2.234) + \frac{(1423 + 602.72)12}{54083.9} = 1.117 + 0.449 = 1.566 \text{ ksi}$$

Allowable compression stress limit for this load combinations = 0.4f'$_c$

f'$_c$ reqd. = 1566/0.4 = 3915 psi

Effective prestress + permanent and transient loads, Service I:

$$f_t = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}} + \frac{(M_{LT} + M_{LL})}{S_{tg}}$$

$$= 2.234 + \frac{(1423 + 602.72)12}{54083.9} = 2.234 + 0.449 = 2.683 \text{ ksi}$$

Allowable compression stress limit for this load combinations = 0.6f'$_c$

f'$_c$ reqd. = 2683/0.6 = 4471.67 psi

Bottom fiber stress in concrete at midspan, Service III

$$f_{bf} = \frac{P_{pe}}{A} + \frac{P_{pe}\,e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1183.46}{788.4} + \frac{19.29(1183.46)}{10521.33} - 4.126 = 1.501 + 2.170 - 4.126 = -0.455 \text{ ksi}$$

Allowable tension in concrete = $0.19\sqrt{f'_c}$

$$f'_{c\ \text{reqd.}} = 1000\left(\frac{0.455}{0.19}\right)^2 = 5734.76 \text{ psi} \qquad \text{(controls)}$$

A.2.7.2.8 Initial Stresses at Hold down point

P_i = 52(0.153)(183.05) = 1456.35 Kips

Initial concrete stress at top fiber of the beam at hold down point

$$f_{ti} = \frac{P_i}{A} - \frac{P_i e_c}{S_t} + \frac{M_g}{S_t}$$

where M_g = moment due to beam self weight at hold down point = 1197.87 K-ft.

$$f_{ti} = \frac{1456.35}{788.4} - \frac{19.29(1456.35)}{8902.67} + \frac{1197.87(12)}{8902.67} = 1.847 - 3.155 + 1.615 = 0.307 \text{ ksi}$$

$$f_{bi} = \frac{P_i}{A} + \frac{P_i e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1456.35}{788.4} + \frac{19.29(1456.35)}{10521.33} - \frac{1197.87(12)}{10521.33} = 1.847 + 2.670 - 1.366 = 3.151 \text{ ksi}$$

Compression stress limit at hold down point = $0.6f'_{ci}$

$$f'_{ci\ reqd.} = \frac{3151}{0.6} = 5251.67 \text{ psi}$$

A.2.7.2.9 Initial Stresses at End

Assuming 10 web strands are draped to top location (5 rows)

$$e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 4(10) + 2(52 + 50 + 48 + 46 + 44)}{52} = 11.21 \text{ in.}$$

$$f_{ti} = \frac{P_i}{A} - \frac{P_i e_e}{S_t}$$

$$f_{ti} = \frac{1456.35}{788.4} - \frac{11.21(1456.35)}{8902.67} = 1.847 - 1.834 = 0.013 \text{ ksi}$$

$$f_{bi} = \frac{P_i}{A} + \frac{P_i e_e}{S_b}$$

$$f_{bi} = \frac{1456.35}{788.4} + \frac{11.21(1456.35)}{10521.33} = 1.847 + 1.552 = 3.399 \text{ ksi}$$

Compression stress limit = $0.6f'_{ci}$

$$f'_{ci\ reqd.} = \frac{3399}{0.6} = 5665 \text{ psi} \qquad \text{(controls)}$$

A.2.7.3
Iteration 3

Number of strands = 52

A.2.7.3.1
Elastic Shortening

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Art. 5.9.5.2.3a]

where

f_{cgp} = sum of concrete stresses at the center of gravity of the prestressing steel due to prestressing force at transfer and self weight of the member at sections of maximum moment

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

where

P_i = pretension force after allowing for the initial losses, assuming 9.61% initial losses = (number of strands)(area of each strand)[0.9039(f_{pi})]
= 52(0.153)(0.9039)(202.5) = 1456.26 Kips

M_g = Unfactored bending moment due to beam self weight = 1209.98 K-ft.

e_c = eccentricity of the strand at the midspan = 19.29 in.

$$f_{cgp} = \frac{1456.26}{788.4} + \frac{1456.26(19.29)^2}{260403} - \frac{1209.98(12)(19.29)}{260403}$$

= 1.847 + 2.081 – 1.076 = 2.852 ksi

E_{ci} = modulus of elasticity of beam at release = $(w_c)^{1.5}(33)\sqrt{f'_{ci}}$

where [LRFD Eq. 5.4.2.4-1]

w_c = weight of concrete = 150 pcf

f'_{ci} = 5665 psi

$$E_{ci} = (150)^{1.5}(33)\sqrt{5665}\ \frac{1}{1000} = 4563 \text{ ksi}$$

E_p = modulus of elasticity of prestressing reinforcement = 28500 ksi

Therefore loss due to elastic shortening:

$$\Delta f_{pES} = \frac{28500}{4563}(2.852) = 17.81 \text{ ksi}$$

A.2.7.3.2
Shrinkage

Δf_{pSR} = 17 – 0.15 H [LRFD Eq. 5.9.5.4.2-1]

where H is relative humidity = 60%

Δf_{pSR} = [17 – 0.15(60)] = 8.0 ksi

A.2.7.3.3
Creep of Concrete

$\Delta f_{pCR} = 12f_{cgp} - 7\Delta f_{cdp}$ [LRFD Eq. 5.9.5.4.3-1]

where

Δf_{cdp} = change of stresses at the center of gravity of the prestressing steel due to permanent loads except the dead load present at the time the prestress force is applied calculated at the same section as f_{cgp}

$$\Delta f_{cdp} = \frac{M_S e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

where

M_S = slab moment = 1179.03 K-ft.

M_{SDL} = superimposed dead load moment = 347.81 K-ft.

y_{bc} = 41.157 in.

y_{bs} = the distance from center of gravity of the strand at midspan to the bottom of the beam = 24.75 – 19.29 = 5.46 in.

I = moment of inertia of the non-composite section =260403 in.^{4}

I_c = moment of inertia of composite section = 694599.5 in.^{4}

$$\Delta f_{cdp} = \frac{1179.03(12)(19.29)}{260403} + \frac{(347.81)(12)(41.157 - 5.46)}{694599.5}$$

Δf_{cdp} = 1.048 + 0.214 = 1.262 ksi

Δf_{pCR} = 12(2.852) – 7(1.262) = 25.39 ksi.

A.2.7.3.4
Relaxation of Prestressing Strands

A.2.7.3.4.1
Relaxation before Transfer

Loss due to Relaxation of steel before transfer

$\Delta f_{pR1} = 0$

A.2.7.3.4.2
Relaxation after Transfer

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

Δf_{pR2} = 30% [20.0 – 0.4 Δf_{pES} – 0.2(Δf_{pSR} + Δf_{pCR})

= 0.3[20.0 – 0.4(17.81) – 0.2(8 + 25.39)] = 1.859 ksi

TxDOT Bridge Design Manual (Pg. # 7-85) recommends that 50% of the final steel relaxation loss be considered for calculation of total initial loss given as [Elastic shortening loss + 0.50(total steel relaxation loss)]

$$\text{Initial Prestress loss \%} = \frac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_{pi}}$$

$= \dfrac{[17.81 + 0.5(1.859)]100}{202.5} = 9.25\% < 9.61\%$ (assumed initial prestress losses)

Therefore next trial is required assuming 9.25% initial losses

$\Delta f_{pES} = \dfrac{E_p}{E_{ci}} f_{cgp}$ [LRFD Art. 5.9.5.2.3a]

where

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

P_i = pretension force after allowing for the initial losses, assuming 9.25% initial losses = (number of strands)(area of each strand)[0.9075(f_{pi})]
= 52(0.153)(0.9075)(202.5) = 1462.06 Kips

M_g = 1209.98 K-ft.

e_c = 19.29 in.

$$f_{cgp} = \frac{1462.06}{788.4} + \frac{1462.06(19.29)^2}{260403} - \frac{1209.98(12)(19.29)}{260403}$$

= 1.854 + 2.089 – 1.076 = 2.867 ksi

E_{ci} = 4563 ksi

E_p = 28500 ksi

Therefore loss due to elastic shortening:

$\Delta f_{pES} = \dfrac{28500}{4563}(2.867) = 17.91$ ksi

$\Delta f_{pCR} = 12f_{cgp} - 7\Delta f_{cdp}$ [LRFD Eq. 5.9.5.4.3-1]

where

f_{cgp} = 2.867 ksi

Δf_{cdp} = 1.262 ksi

Δf_{pCR} = 12(2.867) – 7(1.262) = 25.57 ksi.

Δf_{pSR} = 8.0 ksi

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

$\Delta f_{pR2} = 30\% [20.0 - 0.4\, \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$

= 0.3[20.0 – 0.4(17.91) – 0.2(8 + 25.57)] = 1.837 ksi

Initial Prestress loss % = $\dfrac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_{pi}}$

$= \dfrac{[17.91 + 0.5(1.837)]100}{202.5} = 9.30\% \approx 9.25\ \%$ (assumed initial prestress losses)

A.2.7.3.5 Total Losses at Transfer

Total Initial loss, $\Delta f_{pi} = (\Delta f_{pES} + 0.5\Delta f_{pR2}) = [17.91 + 0.5(1.837)] = 18.83$ ksi

f_{pi} = effective initial prestress = 202.5 – 18.83 = 183.67 ksi

P_i = effective pretension force after allowing for the initial losses

= 52(0.153)(183.67) = 1461.28 Kips

A.2.7.3.6 Total Losses at Service Loads

Δf_{pES} = 17.910 ksi

Δf_{pSR} = 8.000 ksi

Δf_{pCR} = 25.570 ksi

Δf_{pR2} = 1.837 ksi

Total final loss, $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}$ [LRFD Eq. 5.9.5.1-1]

= 17.910 + 8.000 + 25.570 + 1.837 = 53.32 ksi

or $\dfrac{53.32(100)}{202.5} = 26.33$ %

f_{pe} = effective final prestress = $f_{pi} - \Delta f_{pT}$ = 202.5 – 53.32 = 149.18 ksi

Check Prestressing stress limit at service limit state:

$f_{pe} \leq 0.8f_{py}$ = 0.8(243) = 194.4 ksi > 149.18 ksi (O.K.)

Total prestressing force after all losses

P_{pe} = 52(0.153)(149.18) = 1186.88 Kips

A.2.7.3.7 Final Stresses at midspan

Concrete stress at the top fiber of the beam, f_t

Effective prestress + permanent loads, Service I:

$$f_t = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$= \frac{1186.88}{788.4} - \frac{19.29(1186.88)}{8902.67} + \frac{(1209.98 + 1179.03)(12)}{8902.67} + \frac{347.81(12)}{54083.9}$$

= 1.505 - 2.572 + 3.22 + 0.077 = 2.23 ksi

Allowable compression stress limit for this load combination = $0.45f'_c$

f'_c reqd. = 2230/0.45 = 4955.55 psi

½(Effective prestress + permanent loads) + transient loads, Service I:

$$f_t = 0.5\left(\frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}\right) + \frac{(M_{LT} + M_{LL})}{S_{tg}}$$

$$= 0.5(2.230) + \frac{(1423 + 602.72)12}{54083.9} = 1.115 + 0.449 = 1.564 \text{ ksi}$$

Allowable compression stress limit for this load combinations = $0.4f'_c$

f'_c reqd. = 1564/0.4 = 3910 psi

Effective prestress + permanent loads + transient loads, Service I:

$$f_t = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}} + \frac{(M_{LT} + M_{LL})}{S_{tg}}$$

$$= 2.230 + \frac{(1423 + 602.72)12}{54083.9} = 2.230 + 0.449 = 2.679 \text{ ksi}$$

Allowable compression stress limit for this load combinations = $0.6f'_c$

f'_c reqd. = 2679/06 = 4465 psi

Bottom fiber stress in concrete at midspan, Service III

$$f_{bf} = \frac{P_{pe}}{A} + \frac{P_{pe}\,e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1186.88}{788.4} + \frac{19.29(1186.88)}{10521.33} - 4.126 = 1.505 + 2.176 - 4.126 = -0.445 \text{ ksi}$$

Allowable tension in concrete = $0.19\sqrt{f'_c}$

$$f'_c \text{ reqd.} = 1000\left(\frac{0.445}{0.19}\right)^2 = 5485.46 \text{ psi} \qquad \text{(controls)}$$

A.2.7.3.8
Initial Stresses at Hold down point

P_i = 52(0.153)(183.67) = 1461.28 Kips

Initial concrete stress at top fiber of the beam at hold down point

$$f_{ti} = \frac{P_i}{A} - \frac{P_i\,e_c}{S_t} + \frac{M_g}{S_t}$$

where M_g = moment due to beam self weight at hold down point = 1197.87 K-ft.

$$f_{ti} = \frac{1461.28}{788.4} - \frac{19.29(1461.28)}{8902.67} + \frac{1197.87(12)}{8902.67} = 1.853 - 3.166 + 1.615 = 0.302 \text{ ksi}$$

$$f_{bi} = \frac{P_i}{A} + \frac{P_i\,e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1461.28}{788.4} + \frac{19.29(1461.28)}{10521.33} - \frac{1197.87(12)}{10521.33} = 1.853 + 2.679 - 1.366 = 3.166 \text{ ksi}$$

Compression stress limit at hold down point = $0.6f'_{ci}$

$$f'_{ci\ reqd.} = \frac{3166}{0.6} = 5276.67 \text{ psi}$$

A.2.7.3.9 Initial Stresses at End

Assuming 10 web strands are draped to top location (5 rows)

$$e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 4(10) + 2(52 + 50 + 48 + 46 + 44)}{52} = 11.21 \text{ in.}$$

$$f_{ti} = \frac{P_i}{A} - \frac{P_i\, e_e}{S_t}$$

$$f_{ti} = \frac{1461.28}{788.4} - \frac{11.21(1461.28)}{8902.67} = 1.853 - 1.840 = -0.013 \text{ ksi}$$

Concrete stresses at bottom fiber

$$f_{bi} = \frac{P_i}{A} + \frac{P_i\, e_e}{S_b}$$

$$f_{bi} = \frac{1461.28}{788.4} + \frac{11.21(1461.28)}{10521.33} = 1.853 + 1.557 = 3.410 \text{ ksi}$$

Compression stress limit = $0.6f'_{ci}$

$$f'_{ci} \text{ reqd.} = \frac{3410}{0.6} = 5683.33 \text{ psi} \qquad \text{(controls)}$$

Therefore provide f'_{ci} = 5683.33 psi

f'_c = 5683.33 psi (f'_c should be atleast equal to f'_{ci})

52 – ½ in. diameter, 10 harped at the end, GR 270 low relaxation strands

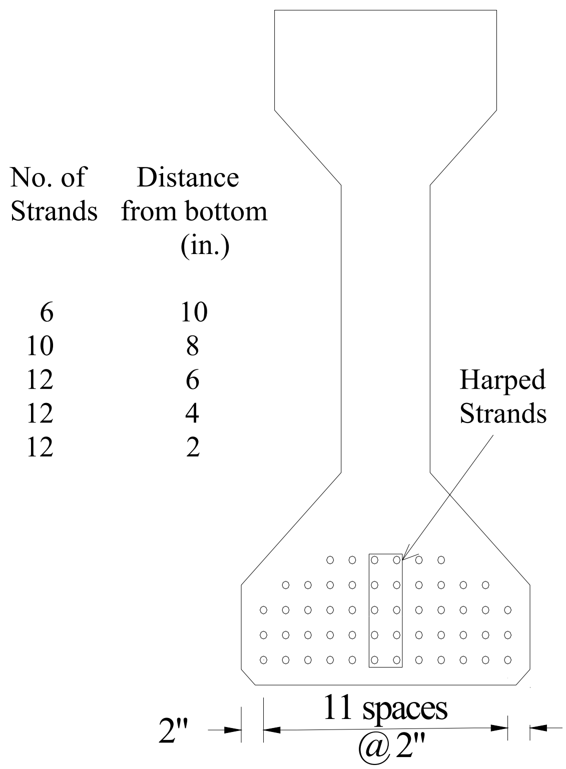

Fig. A.2.7.1
Final Strand Pattern at Midspan

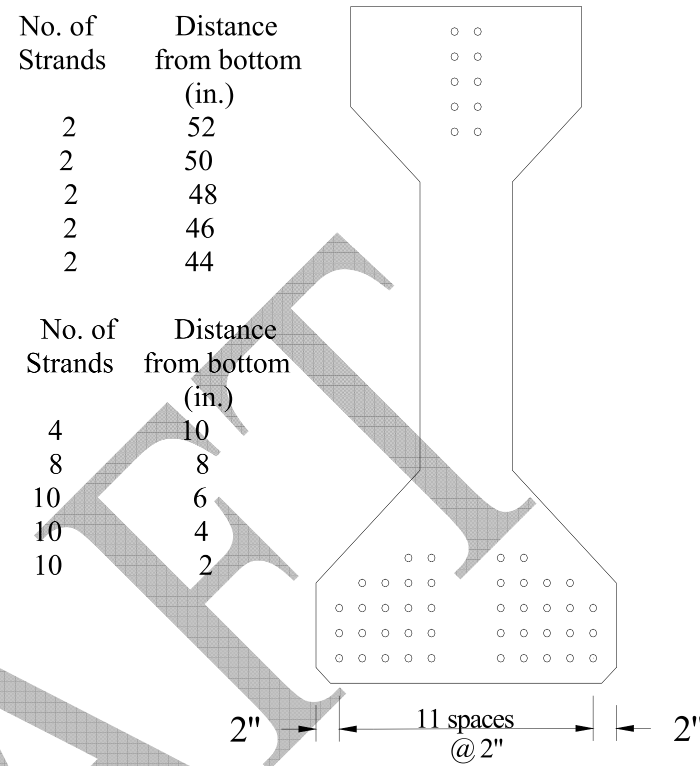

Fig. A.2.7.2
Final Strand Pattern at Ends

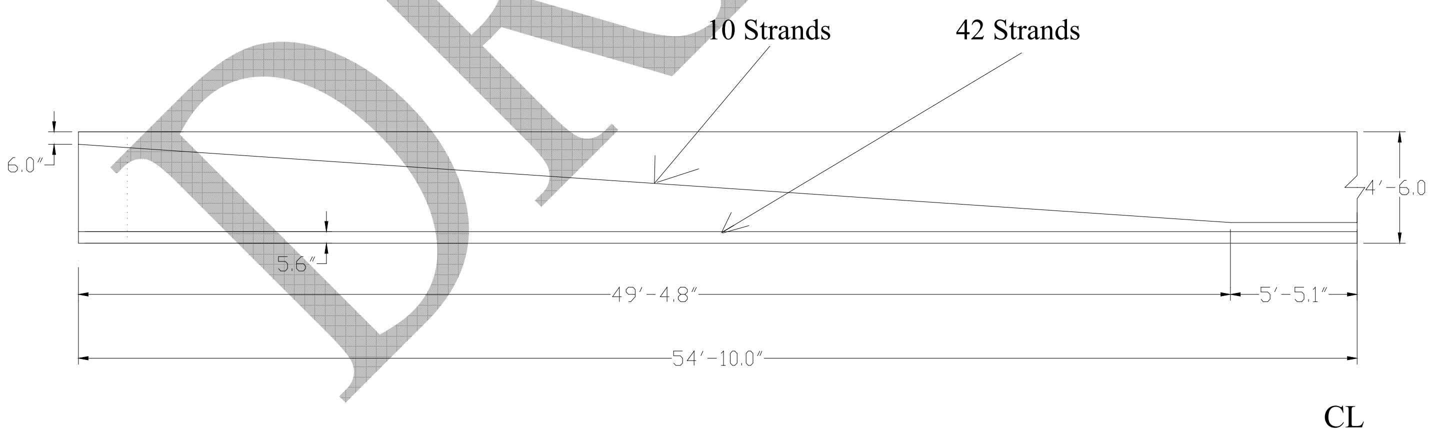

Fig. A.2.7.3 Longitudinal strand profile (Half of the beam length is shown)

Note: The hold down distance of 49.4 ft. is measured from the end of the beam.

The distance between the center of gravity of the 10 harped strands and the top of the beam at the end of the beam = $\dfrac{2(2) + 2(4) + 2(6) + 2(8) + 2(10)}{10}$ = 6 in.

The distance between the center of gravity of the 10 harped strands and the bottom of the beam at harp points = $\dfrac{2(2) + 2(4) + 2(6) + 2(8) + 2(10)}{10}$ = 6 in.

The distance between the center of gravity of the 10 harped strands and the top of the beam at transfer length section

$$= 6 \text{ in.} + \frac{(54 \text{ in} - 6 \text{ in} - 6 \text{ in})}{49.4 \text{ ft.}}(2.5 \text{ ft.}) = 8.12 \text{ in.}$$

The distance between the center of gravity of the 42 straight strands and the bottom of the beam at all locations

$$y_{bsend} = \frac{10(2) + 10(4) + 10(6) + 8(8) + 4(10)}{42} = 5.33 \text{ in.}$$

A.2.8 CHECK FOR DISTRIBUTION FACTORS

The distribution factor for moment based on actual modular ratio is calculated and compared to the one used in the design until this point. If the change is found to be large then the bending moments and Shear forces will be updated based on new Distribution factors and used for further design.

A.2.8.1 Distribution factor for Bending Moment

For all limit states except fatigue limit state:

For two or more lanes loaded:

$$DFM = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$ [LRFD Table 4.6.2.2.2b-1]

Provided that: $3.5 \leq S \leq 16$; S = 8.0 ft (O.K.)

$4.5 \leq t_s \leq 12$; t_s = 8.0 in (O.K.)

$20 \leq L \leq 240$; L = 108.583 ft. (O.K.)

$N_b \geq 4$; $N_b = 6$ (O.K.)

$10{,}000 \leq K_g \leq 7{,}000{,}000$ (O.K., as shown below)

where

DFM = distribution factor for moment for interior beam

S = spacing of beams = 8.0 ft

L = beam span = 108.583 ft

t_s = depth of concrete deck = 8.0 in.

K_g = longitudinal stiffness parameter, in.^{4}

$K_g = n\,(I + Ae_g^2)$ [LRFD Art. 3.6.1.1.1]

where

A = cross-sectional area of the beam (non-composite section)

$A = 788.4$ in.^{2}

I = moment of inertia of beam (non-composite section) = 260,403.0 in.^{4}

e_g = distance between centers of gravity of the beam and slab, in.

$= (t_s/2 + y_t) = (8/2 + 29.25) = 33.25$ in.

n = modular ratio between beam and slab materials = $\dfrac{E_c(\text{beam})}{E_c(\text{deck})}$

E_c(beam) = modulus of elasticity of beam at service = $(w_c)^{1.5}(33)\sqrt{f'_c}$

where [LRFD Eq. 5.4.2.4-1]

w_c = weight of beam concrete = 150 pcf

f'_c = 5683.33 psi

$$E_c(\text{beam}) = (150)^{1.5}(33)\sqrt{5683.33}\ \frac{1}{1000} = 4570.38 \text{ ksi}$$

E_c(deck) = modulus of elasticity of slab = $(w_c)^{1.5}(33)\sqrt{f'_c}$

where [LRFD Eq. 5.4.2.4-1]

w_c = weight of slab concrete = 150 pcf

f'_c = 4000.0 psi

$$E_c(\text{deck}) = (150)^{1.5}(33)\sqrt{4000}\ \frac{1}{1000} = 3834.25 \text{ ksi}$$

$$n = \frac{4570.38}{3834.25} = 1.192$$

Therefore

$K_g = 1.192[260403 + 788.4(33.25)^2] = 1{,}349{,}377.94$ in.^{4}

$$\text{DFM} = 0.075 + \left(\frac{8}{9.5}\right)^{0.6}\left(\frac{8}{108.583}\right)^{0.2}\left(\frac{1349377.94}{12.0(108.583)(8)^3}\right)^{0.1}$$

$$= 0.075 + (0.902)(0.593)(1.073) = 0.649 \text{ lanes/beam}$$

For one design lane loaded:

$$\text{DFM} = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$

$$= 0.06 + \left(\frac{8}{14}\right)^{0.4}\left(\frac{8}{108.583}\right)^{0.3}\left(\frac{1349377.94}{12.0(108.583)(8)^3}\right)^{0.1}$$

$$= 0.06 + (0.8)(0.457)(1.073) = 0.452 \text{ lanes/beam}$$

Thus, the case of two or more lanes loaded controls

DFM = 0.649 lanes/beam

For fatigue limit state:

LRFD Specifications, Art. 3.4.1, states that for fatigue limit state, a single design truck should be used. However, live load distribution factors given in LRFD Article 4.6.2.2 take into consideration the multiple presence factor, m. LRFD Article 3.6.1.1.2 states that the multiple presence factor, m, for one design lane loaded is 1.2. Therefore, the distribution factor for one design lane loaded with the multiple presence factor removed should be used. The distribution factor for the fatigue limit state is: 0.452/1.2 = 0.377 lanes/beam.

$$\text{Percent change in DFM} = \left(\frac{0.649 - 0.639}{0.649}\right)100 = 1.54\%$$

$$\text{Percent change in DFM for fatigue limit state} = \left(\frac{0.377 - 0.371}{0.377}\right)100 = 1.6\%$$

The change in the distribution factors is very less and can be neglected as they won't impact the moments by a great amount. The moments obtained using previous distribution factors are used in the following design.

STRESS SUMMARY

A.2.8.1 Concrete Stresses at Transfer

A.2.8.1.1 Allowable Stress limits

[LRFD Art. 5.9.4]

Compression: 0.6 f'_{ci} = 0.6(5683.33) = +3410.0 psi = 3.410 ksi (compression)

Tension:

Without bonded reinforcement

$0.0948\sqrt{f'_{ci}} \leq 0.200$ ksi; $-0.0948\sqrt{5.683} = -0.226$ ksi

Therefore –0.200 ksi controls

With bonded auxiliary reinforcement which is sufficient to resist 120% of the tension force in the cracked concrete

$-0.24\sqrt{f'_{ci}} = -0.24\sqrt{5.683} = -0.572$ ksi

A.2.8.1.2 Stresses at Beam End

Total prestressing force after transfer = 52(0.153)(183.67) = 1461.28 kips.

Stresses at end are checked only at release, because it almost always governs. Assuming all web strands are draped to top location (5 rows)

Eccentricity of strands at the end of the beam

$$e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 4(10) + 2(52 + 50 + 48 + 46 + 44)}{52} = 11.21 \text{ in.}$$

End concrete stress at top fiber at release

$$f_{ti} = \frac{P_i}{A} - \frac{P_i e_e}{S_t}$$

$$f_{ti} = \frac{1461.28}{788.4} - \frac{11.21(1461.28)}{8902.67} = 1.853 - 1.84 = 0.013 \text{ ksi}$$

Allowable compression: +3.410 ksi >> 0.013 ksi (reqd.) (O.K.)

There is no need for additional bonded reinforcement as there is no tension in the top fiber of the beam.

End concrete stress at bottom fiber at release

$$f_{bi} = \frac{P_i}{A} + \frac{P_i e_e}{S_b}$$

$$f_{bi} = \frac{1461.28}{788.4} + \frac{11.21(1461.28)}{10521.33} = 1.853 + 1.557 = 3.410 \text{ ksi}$$

Allowable compression: +3.410 ksi = 3.410 ksi (reqd.) (O.K.)

A.2.8.1.3 Stresses at Transfer Length Section

Stresses at transfer length are checked only at release, because it almost always governs.

Transfer length = 60(strand diameter) [LRFD Art. 5.8.2.3]

= 60 (0.50) = 30 in. = 2.5 ft.

Transfer length section is located at a distance of 2.5 ft. from end of the beam or at a point 1.96 ft. from center of the bearing as the beam extends 6.5 in. beyond the bearing centerline. Overall beam length of 109.67 ft. is considered for the calculation of bending moment at transfer length.

Moment due to beam self weight, $M_g = 0.5wx\,(L\text{-}x)$

$$M_g = 0.5(0.821)(2.5)(109.67 - 2.5)$$

$$= 110.00 \text{ K-ft.}$$

Concrete stress at top fiber of the beam

$$f_t = \frac{P_i}{A} - \frac{P_i e_t}{S_t} + \frac{M_g}{S_t}$$

Strand eccentricity at transfer section, $e_x = e_c - (e_c - e_e)\dfrac{(48.862 - x)}{48.862}$

$$e_t = 19.29 - (19.29 - 11.21)\frac{(48.862 - 2.5)}{48.862} = 11.62 \text{ in.}$$

$$f_t = \frac{1461.28}{788.4} - \frac{11.62(1461.28)}{8902.67} + \frac{110.00(12)}{8902.67} = 1.853 - 1.907 + 0.148 = 0.094 \text{ ksi}$$

Allowable compression: 3.410 ksi >> 0.094 ksi (reqd.) (O.K.)

Concrete stress at the bottom fiber of the beam

$$f_b = \frac{P_i}{A} + \frac{P_i e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1461.28}{788.4} + \frac{11.62(1461.28)}{10521.33} - \frac{110.00(12)}{10521.33} = 1.853 + 1.614 - 0.125 = 3.342 \text{ ksi}$$

Allowable compression: +3.410 ksi > 3.342 ksi (reqd.) (O.K.)

A.2.8.1.4
Stresses at Harp points

Eccentricity of the strands at harp points is same as at midspan

$e_{harp} = e_c = 19.29$ in.

Distance of harp point from bearing center line = 48.862 ft.

Distance of harp point from beam end = (6.5/12) + 48.862 = 49.4 ft.

Bending moment at harp point due to beam self weight based on overall beam length, $M_g = 0.5wx\,(L\text{-}x) = 0.5(0.821)(49.4)(109.67 - 49.4) = 1222.2$ K-ft.

Concrete stress at top fiber of the beam at hold down point

$$f_t = \frac{P_i}{A} - \frac{P_i e_c}{S_t} + \frac{M_g}{S_t}$$

$$f_t = \frac{1461.28}{788.4} - \frac{19.29(1461.28)}{8902.67} + \frac{1222.2(12)}{8902.67} = 1.853 - 3.166 + 1.647 = 0.334 \text{ ksi}$$

Allowable compression: +3.410 ksi >> 0.334 ksi (Reqd.) (O.K.)

Concrete stresses in bottom fibers of the beam at hold down point

$$f_b = \frac{P_i}{A} + \frac{P_i e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_b = \frac{1461.28}{788.4} + \frac{19.29(1461.28)}{10521.33} - \frac{1222.2(12)}{10521.33} = 1.853 + 2.679 - 1.394 = 3.138 \text{ ksi}$$

Allowable compression: +3.410 ksi > 3.138 ksi (reqd.) (O.K.)

A.2.8.1.5
Stresses at Midspan

Bending moment at midspan due to beam self –weight based on overall length

$M_g = 0.5wx\,(L\text{-}x)$

$= 0.5(0.821)(54.835)(109.67 – 54.835) = 1234.32$ K-ft.

Concrete stress at top fiber of the beam at midspan

$$f_t = \frac{P_{si}}{A} - \frac{P_{si}\, e_c}{S_t} + \frac{M_g}{S_t}$$

$$f_t = \frac{1461.28}{788.4} - \frac{19.29(1461.28)}{8902.67} + \frac{1234.32(12)}{8902.67} = 1.853 – 3.166 + 1.664 = 0.351 \text{ ksi}$$

Allowable compression: +3.410 ksi >> 0.351 ksi (Reqd.) (O.K.)

Concrete stresses in bottom fibers of the beam at hold down point

$$f_b = \frac{P_{si}}{A} + \frac{P_{si}\, e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_b = \frac{1461.28}{788.4} + \frac{19.29(1461.28)}{10521.33} - \frac{1234.32(12)}{10521.33} = 1.853 + 2.679 - 1.408 = 3.124 \text{ ksi}$$

Allowable compression: +3.410 ksi > 3.124 ksi (reqd.) (O.K.)

A.2.8.1.6
Stress Summary
at transfer

	Top of beam f_t (ksi)	Bottom of beam f_b (ksi)
At End	+0.013	+3.410
At transfer length section	+0.094	+3.342
At harp Points	+0.334	+3.138
At Midspan	+0.351	+3.124

A.2.8.2
Concrete Stresses
at Service Loads

A.2.8.2.1
Allowable Stress Limits

[LRFD Art. 5.9.4.2]

Compression

Due to (Effective prestress + permanent loads) for load combination service I

$0.45f'_c = 0.45(5683.33)/1000 = +2.557$ ksi (for precast beam)

$0.45f'_c = 0.45(4000)/1000 = +1.800$ ksi (for slab)

Due to ½(effective prestress + permanent loads) + transient loads for load combination Service I

$0.40f'_c = 0.40(5683.33)/1000 = +2.273$ ksi (for precast beam)

Due to permanent and transient loads for load combination Service I

$0.60f'_c = 0.60(5683.33)/1000 = +3.410$ ksi (for precast beam)

$0.60f'_c = 0.60(4000)/1000 = +2.400$ ksi (for slab)

Tension:

For components with bonded prestressing tendons

for load combination Service III: $-0.19\sqrt{f'_c}$

for precast beam: $-0.19\sqrt{5.683} = -0.453$ ksi

A.2.8.2.2 Stresses at Midspan

$P_{pe} = 52(0.153)(149.18) = 1186.88$ Kips

Concrete stresses at top fiber of the beam:

The compressive stresses are checked for two cases

1. Effective prestress + permanent loads, Service I

$$f_t = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_g+M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$f_t = \frac{1186.88}{788.4} - \frac{19.29(1186.88)}{8902.67} + \frac{(1209.98+1179.03)(12)}{8902.67} + \frac{(347.81)(12)}{54083.9}$$

$f_t = 1.505 - 2.572 + 3.220 + 0.077 = 2.230$ ksi

Allowable compression: +2.557 ksi > +2.230 ksi (reqd.) (O.K.)

2. ½(Effective prestress + permanent loads) + transient loads, Service I:

$$f_t = 0.5\left(\frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_g+M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}\right) + \frac{M_{LL}+M_{LT}}{S_{tg}}$$

$$0.5\left(\frac{1186.88}{788.4} - \frac{19.29(1186.88)}{8902.67} + \frac{(1209.98+1179.03)(12)}{8902.67} + \frac{347.81(12)}{54083.9}\right) + \frac{(1423.00+602.72)12}{54083.9}$$

$f_t = 0.5(1.505 - 2.572 + 3.220 + 0.077) + 0.449 = 1.564$ ksi

Allowable compression: +2.273 ksi > +1.564 ksi (reqd.) (O.K.)

3. Under permanent and transient loads, Service I:

$$f_t = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_g+M_S}{S_t} + \frac{M_{SDL}}{S_{tg}} + \frac{M_{LL}+M_{LT}}{S_{tg}}$$

$$\frac{1186.88}{788.4} - \frac{19.29(1186.88)}{8902.67} + \frac{(1209.98+1179.03)(12)}{8902.67} + \frac{347.81(12)}{54083.9} + \frac{(1423.00+602.72)12}{54083.9}$$

$f_t = 1.505 – 2.572 + 3.220 + 0.077 + 0.449 = 2.679$ ksi

Allowable compression: +3.410 ksi > +2.679 ksi (reqd.) (O.K.)

Stresses at the top of the deck

1. Under permanent loads, Service I

$$f_t = \frac{M_{SDL}}{S_{tc}} = \frac{(347.81)(12)}{33325.31} = 0.125 \text{ ksi}$$

Allowable compression: +1.800 ksi > +0.125 ksi (reqd.) (O.K.)

2. Under permanent and transient loads, Service I

$$f_{tc} = \frac{M_{SDL}+M_{LT}+M_{LL}}{S_{tc}} = \frac{(347.81 + 1423.00 + 602.72)(12)}{33325.31} = +0.855 \text{ ksi}$$

Allowable compression: +2.400 ksi > +0.855 ksi (reqd.) (O.K.)

Concrete stresses at bottom fiber of the beam, Service III:

$$f_b = \frac{P_{pe}}{A} + \frac{P_{pe}\,e_c}{S_b} - \frac{M_g+M_S}{S_b} - \frac{M_{SDL}+0.8(M_{LT}+M_{LL})}{S_{bc}}$$

$$\frac{1186.88}{788.4} + \frac{19.29(1186.88)}{10521.33} - \frac{(1209.98+1179.03)(12)}{10521.33} - \frac{\left[347.81+0.8(1423.00+602.72)\right](12)}{16876.83}$$

$f_b = 1.505 + 2.176 - 2.725 - 1.400 = –0.444$ ksi

Allowable Tension: –0.453 ksi (O.K.)

A.2.8.2.3
Fatigue Stress Limit

In regions of compressive stress due to permanent loads and prestress, fatigue is considered only if the compressive stress is less than twice the maximum tensile live load stress resulting from fatigue load combination [LRFD Art. 5.5.3.1]

At midspan, bottom fiber stress due to permanent loads and prestress is:

$$\frac{P_{pe}}{A} + \frac{P_{pe}\,e_c}{S_b} - \frac{M_g+M_S}{S_b} - \frac{M_{SDL}}{S_{bc}}$$

$$= \frac{1186.88}{788.4} + \frac{19.29(1186.88)}{10521.33} - \frac{(1209.98+1179.03)(12)}{10521.33} - \frac{347.81(12)}{16876.83}$$

= 1.505 + 2.176 – 2.725 – 0.247 = 0.709 ksi

Stress at the bottom fiber of the beam due to fatigue load combination is:

$$-\frac{0.75(M_f)}{S_{bc}} = -\frac{0.75(605.16)(12)}{16876.83} = -0.323 \text{ ksi}$$

2(0.323 ksi) = 0.646 ksi < 0.709 ksi, therefore fatigue check need not be performed.

A.2.8.2.4 Summary of Stresses at Service loads

	Top of Deck (ksi) Service I		Top of Beam (ksi) Service I		Bottom of Beam (ksi) Service III
	Permanent loads	Total Loads	Permanent loads	Total Loads	
At Midspan	+0.125	+0.855	+2.230	+2.679	–0.444

A.2.9 STRENGTH LIMIT STATE

Total ultimate bending moment for Strength I is:

M_u = 1.25(DC) + 1.5(DW) + 1.75(LL + IM)

where

DC = bending moment due to dead loads except wearing surface

DW = bending moment due to wearing surface load

LL+IM = bending moment due to live load and impact

The ultimate bending moment at midspan is

M_u = 1.25(1209.98 + 1179.03 + 162.12) + 1.5(185.70) + 1.75(1423 + 602.72)

= 7012.47 Kip-ft.

f_{pe} = 149.18 ksi > $0.5f_{pu}$ = 0.5(270) = 135 ksi

Average stress in pretensioning steel when $f_{pe} \geq 0.5f_{pu}$:

$$f_{ps} = f_{pu}\left(1\text{-}k\frac{c}{d_p}\right)$$ [LRFD Eq. 5.7.3.1.1-1]

where

f_{ps} = average stress in prestressing steel

f_{pu} = specified tensile strength of prestressing steel = 270 ksi

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right)$$ [LRFD Eq. 5.7.3.1.1-2]

= 0.28 for low-relaxation strand [LRFD Table C5.7.3.1.1-1]

d_p = distance from extreme compression fiber to the centroid of the prestressing tendons = $h - y_{bs}$ = 62 – 5.46 = 56.54 in.

c = distance between neutral axis and the compressive face, in.

To compute c, assume rectangular section behavior and check if the depth of the equivalent compression stress block, a, is less than or equal to t_s [LRFD C5.7.3.2.2]

where $a = \beta_1 c$

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A_s'f_y'}{0.85f_c'\beta_1 b + kA_{ps}\frac{f_{pu}}{d_p}}$$ [LRFD Eq. 5.7.3.1.1.-4]

where

A_{ps} = area of prestressing steel = 52(0.153) = 7.956 in.^{2}

A_s = area of mild steel tension reinforcement = 0 in.^{2}

A_s’ = area of compression reinforcement = 0 in.^{2}

f_c' = compressive strength of deck concrete = 4 ksi

f_y = yield strength of tension reinforcement, ksi

f_y' = yield strength of compression reinforcement, ksi

β_1 = stress factor of compression block [LRFD Art. 5.7.2.2]

= 0.85 for $f_c' \leq 4.0$ ksi

b = effective width of compression flange = 96 in.

$$c = \frac{7.956(270) + 0 - 0}{0.85(4.0)(0.85)(96) + 0.28(7.956)\left(\frac{270}{56.54}\right)} = 7.457 \text{ in.} < t_s = 8.0 \text{ in.} \quad \text{O.K.}$$

$a = \beta_1 c = 0.85(7.457) = 6.338$ in.$< t_s = 8$ in.

Therefore, the rectangular section behavior assumption is valid

The average stress in prestressing steel is:

$$f_{ps} = 270\left(1 - 0.28\frac{7.457}{56.54}\right) = 260.03 \text{ ksi}$$

Nominal flexural resistance: [LRFD Art. 5.7.3.2.3]

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) \quad \text{[LRFD Eq. 5.7.3.2.2-1]}$$

The above equation is a simplified form of LRFD Equation 5.7.3.2.2-1 because no compression reinforcement or mild tension reinforcement is considered and the section behaves as a rectangular section

$$M_n = (7.956)(260.03)\left(56.54 - \frac{6.338}{2}\right)/12 = 9201.15 \text{ kip-ft.}$$

Factored flexural resistance:

$M_r = \phi M_n$ [LRFD Eq. 5.7.3.2.1-1]

where

ϕ = resistance factor [LRFD Art. 5.5.4.2.1]

= 1.0 for flexure and tension of prestressed concrete

$M_r = 1(9201.15) = 9201.15$ kip-ft. $> M_u =$ 7012.47 kip-ft O.K.

A.2.10 LIMITS OF REINFORCEMENT

A.2.10.1 Maximum Reinforcement

[LRFD Art. 5.7.3.3.1]

The amount of prestressed and non-prestressed reinforcement should be such that

$$\frac{c}{d_e} \le 0.42 \qquad \text{[LRFD Eq. 5.7.3.3.1-1]}$$

where $d_e = \dfrac{A_{ps}f_{ps}d_p + A_sf_yd_s}{A_{ps}f_{ps} + A_sf_y}$ [LRFD Eq. 5.7.3.3.1-2]

$A_s = 0$, Therefore $d_e = d_p = 56.54$ in.

$$\frac{c}{d_e} = \frac{7.457}{56.54} = 0.132 < 0.42 \qquad \text{O.K.}$$

A.2.10.2 Minimum Reinforcement

[LRFD Art. 5.7.3.3.2]

At any section, the amount of prestressed and non-prestressed tensile reinforcement should be adequate to develop a factored flexural resistance, M_r, equal to the lesser of:

1.2 times he cracking strength determined on the basis of elastic stress distribution and the modulus of rupture, and

1.33 times the factored moment required by the applicable strength load combination.

Check at midspan:

Cracking moment:

$$M_{cr} = (f_r + f_{pe})\, S_{bc} - M_{d\text{-}nc}\left(\frac{S_{bc}}{S_b} - 1\right) \le S_c f_r \qquad \text{[LRFD Eq. 5.7.3.3.2-1]}$$

where

f_r = modulus of rupture [LRFD Art. 5.4.2.6]

$= 0.24\sqrt{f'_c} = 0.24\sqrt{5.683} = 0.572$ ksi

f_{pe} = compressive stress in concrete due to effective prestress forces at extreme fiber of section where tensile stress is caused by externally applied loads

$$f_{pe} = \frac{P_{pe}}{A} + \frac{P_{pe}e_c}{S_b}$$

where

P_{pe} = effective prestress force after losses = 1186.88 Kips

e_c = 19.29 in.

$$f_{pe} = \frac{1186.88}{788.4} + \frac{1186.88(19.29)}{10521.33} = 1.505 + 2.176 = 3.681 \text{ ksi}$$

$M_{d\text{-}nc}$ = non-composite dead load moment at midspan due to self weight of beam and weight of slab = 1209.98 + 1179.03 = 2389.01 kip-ft.

S_b = non-composite section modulus for the extreme fiber of the section where the tensile stress is caused by externally applied loads = 10521.33 in.^{4}

S_{bc} = composite section modulus for the extreme fiber of the section where the tensile stress is caused by externally applied loads = 16876.83 in.^{4}

$$= (0.572 + 3.681)(16876.83)\left(\frac{1}{12}\right) - 2389.01\left(\frac{16876.83}{10521.33} - 1\right)$$

= 5981.43 – 1443.10 = 4538.33 kip-ft.

$S_c f_r$ = 16876.83(0.572) = 9653.55 kips-ft. > 4538.33 kip-ft.

M_{cr} = 4538.33 kip-ft.

1.2 M_{cr} = 1.2(4538.33) = 5446.0 kip-ft.

At midspan, factored moment required by Strength I load combination is:

M_u = 7012.47 kip-ft.

$1.33M_u$ = 1.33(7012.47 K-ft.) = 9326.59 kip-ft.

Since $1.2M_{cr} < 1.33M_u$, the $1.2M_{cr}$ requirement controls.

M_r = 9201.15 kip-ft > $1.2M_{cr}$ = 5446.0 kip-ft. O.K.

A.2.11 SHEAR DESIGN

The area and spacing of shear reinforcement must be determined at regular intervals along the entire length of the beam. In this design example, transverse shear design procedures are demonstrated below by determining these values at the critical section near the supports.

Transverse shear reinforcement is provided when:

$V_u < 0.5 \varnothing (V_c + V_p)$ [LRFD Art. 5.8.2.4-1]

where

V_u = total factored shear force at the section, kips

V_c = shear strength provided by concrete

V_s = component of the effective prestressing force in the direction of the applied shear, Kips

$\varnothing$ = resistance factor = 0.90 [LRFD Art. 5.5.4.2.1]

A.2.11.1 Critical Section

Critical section near the supports is the greater of: [LRFD Art. 5.8.3.2]

$0.5d_v\cot\theta$ or d_v

where

d_v = effective shear depth

= distance between resultants of tensile and compressive forces, (d_e-a/2), but not less than the greater of ($0.9d_e$) or (0.72h) [LRFD Art. 5.8.2.9]

where

d_e = the corresponding effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement [LRFD Art. 5.7.3.3.1]

a = depth of compression block = 6.338 in. at midspan

h = total height of section = 62 in.

θ = angle of inclination of diagonal compressive stresses, assume θ is 23^0 (slope of compression field)

A.2.11.1.1 Angle of Diagonal Compressive Stresses

The shear design at any section depends on the angle of diagonal compressive stresses at the section. Shear design is an iterative process that begins with assuming a value for θ.

A.2.11.1.2 Effective Shear Depth

Since some of the strands are harped, the effective depth d_e, varies from point to point. However d_e must be calculated at the critical section in shear which is not yet known; therefore, for the first iteration, d_e is calculated based on the center of gravity of the straight strand group at the end of the beam, y_{bsend}.

$d_e = h_c - y_{bsend}$ = 62.0 – 5.33 = 56.67 in.

dv = d_e – 0.5(a) = 56.67 – 0.5(6.338) = 53.50 in (controls)

$\geq 0.9\, d_e$ = 0.9 (56.67) = 51.00 in

$\geq$ 0.72h = 0.72(62) = 44.64 in O.K.

Therefore d_v = 53.50 in.

A.2.11.1.3 Calculation of critical section

The critical section near the support is greater of:

d_v = 53.50 in

and

$0.5d_v\cot\theta = 0.5(53.50)(\cot 23^0)$ = 63.02 in. from face of the support (controls)

Adding half the bearing width (3.25 in.) to critical section distance from face of the support to get the distance of the critical section from center line of bearing.

Critical section for shear is

63.02 + 3.25 = 66.27 in. = 5.52 ft. from centerline of bearing

x/L = 5.52/108.583 = 0.05L

It is conservative not to refine the value of d_e based on the critical section 0.05L. The value if refined will have small difference.

A.2.11.2
Contribution of Concrete to Nominal Shear Resistance

The contribution of the concrete to the nominal shear resistance is:

$$V_c = 0.0316\beta\sqrt{f'_c}\,b_v d_v \qquad \text{[LRFD Eq. 5.8.3.3-3]}$$

A.2.11.2.1
Strain in Flexural Tension Reinforcement

Calculate the strain in the reinforcement, ε_x on the flexural tension side. Assuming that the section contains at least the minimum transverse reinforcement as specified in LRFD Specifications Art. 5.8.2.5:

$$\varepsilon_x = \frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}}{2(E_sA_s + E_pA_{ps})} \le 0.001 \qquad \text{[LRFD Eq. 5.8.3.4.2-1]}$$

where

V_u = applied factored shear force at the specified section, 0.05L

= 1.25(40.04 + 39.02 + 5.36) +1.50(6.15) + 1.75(67.28 + 25.48)

= 277.08 kips

M_u = applied factored moment at the specified section, 0.05L > $V_u d_v$

= 1.25(233.54 + 227.56 + 31.29) + 1.50(35.84) + 1.75(291.58 + 116.33)

= 1383.09 Kip-ft. > 277.08(53.5/12) = 1253.32 Kip-ft. O.K.

N_u = applied factored normal force at the specified section, 0.05L = 0

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked in difference in strain between the prestressing tendons and the surrounding concrete (ksi) For pretensioned members, LRFD Art. C5.8.3.4.2 indicates that f_{po} can be taken as the stress in strands when the concrete is cast around them, which is jacking stress f_{pj}, or $f_{pu.}$

= 0.75(270.0) = 202.5 ksi

V_p = component of the effective prestressing force in the direction of the applied shear = (Force per strand)(Number of harped strands)(sinΨ)

$$\Psi = \tan^{-1}\left(\frac{42.67}{49.4(12)}\right) = 0.072 \text{ rad.}$$

V_p = 22.82(10) sin (0.072) = 16.42 Kips

$$\varepsilon_x = \frac{\dfrac{1383.09(12)}{53.50} + 0.5(0.0) + 0.5(277.08 - 16.42)\cot 23^{o} - 42(0.153)202.5}{2\left[28000(0.0) + 28500(42)(0.153)\right]} \leq 0.001$$

$\varepsilon_x = -0.00187$

Since this value is negative LRFD Eq. 5.8.3.4.2-3 should be used to calculate ε_x

$$\varepsilon_x = \frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5(V_u\text{-}V_p)\cot\theta - A_{ps}f_{po}}{2(E_cA_c + E_sA_s + E_pA_{ps})}$$

where

A_c = area of the concrete (in.^{2}) on the flexural tension side below h/2

A_c = 473 in.^{2}

$E_c = (150)^{1.5}(33)\sqrt{5683.33}\ \dfrac{1}{1000} = 4570.38$ ksi

$$\varepsilon_x = \frac{\dfrac{1383.09(12)}{53.50} + 0.5(0.0) + 0.5(277.08 - 16.42)\cot 23^{o} - 42(0.153)202.5}{2\left[4570.38(473) + 28000(0.0) + 28500(42)(0.153)\right]}$$

$\varepsilon_x = -0.000146$

$$\upsilon_u = \frac{V_u - \varphi V_p}{\varphi b_v d_v}$$ [LRFD Eq. 5.8.3.4.2-1]

where

υ_u = shear stress in concrete

φ = resistance factor = 0.9 [LRFD Art. 5.5.4.2.1]

b_v = web width = 8 in. [LRFD Art. 5.8.2.9]

$$\upsilon_u = \frac{277.08 - 0.9(16.42)}{0.9(8)(53.50)} = 0.681 \text{ ksi}$$

$\upsilon_u/f'_c = 0.681/5.683 = 0.12$

A.2.11.2.2 Values of β and θ

The values of β and θ are determined using LRFD Table 5.8.3.4.2-1. Linear interpolation is allowed if the values lie between two rows

υ_u/f'_c	ε_x x 1000		
	≤–0.200	–0.146	≤ –0.100
≤ 0.100	18.100		20.400
	3.790		3.380
0.12	19.540	20.65	21.600
	3.302	3.176	3.068
≤ 0.125	19.900		21.900
	3.180		2.990

$\theta = 20.65^0 < 23^0$ (assumed)

Another iteration is made with $\theta = 20.65^0$ to arrive at the correct value of β and θ.

$d_e = 56.67$ in.

$d_v = 53.50$ in.

The critical section near the support is greater of:

$d_v = 53.50$ in

and

$0.5d_v\cot\theta = 0.5(53.50)(\cot 20.65^0) = 71.0$ in. from face of the support (controls)

Adding half the bearing width (3.25 in.) to critical section distance from face of the support to get the distance of the critical section from center line of bearing.

Critical section for shear is

71.0 + 3.25 = 74.25 in. = 6.19 ft. from centerline of bearing

x/L = 6.19/108.583 = 0.057L

It is conservative not to refine the value of d_e based on the critical section 0.057L. The value if refined will have small difference.

Calculate the strain in the reinforcement, ε_x on the flexural tension side. Assuming that the section contains at least the minimum transverse reinforcement as specified in LRFD Specifications Art. 5.8.2.5:

Assuming the strain will be negative again, LRFD Eq. 5.8.3.4.2-3 should be used to calculate ε_x

$$\varepsilon_x = \frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}}{2(E_cA_c + E_sA_s + E_pA_{ps})}$$

where

V_u = applied factored shear force at the specified section, 0.057L

= 1.25(39.49 + 38.48 + 5.29) +1.50(6.06) + 1.75(66.81 + 25.15)

= 274.10 kips

M_u = applied factored moment at the specified section, 0.058L > V_ud_v

= 1.25(260.18 + 253.53 + 34.86) + 1.50(39.93) + 1.75(324.63 + 129.60)

= 1540.50 Kip-ft. > 274.10(53.5/12) = 1222.03 Kip-ft. O.K.

N_u = applied factored normal force at the specified section, 0.057L = 0

f_{po} = 0.75(270.0) = 202.5 ksi

V_p = 22.82(10) sin (0.072) = 16.42 Kips

A_c = 473 in.^{2}

$$E_c = (150)^{1.5}(33)\sqrt{5683.33}\ \frac{1}{1000} = 4570.38 \text{ ksi}$$

$$\varepsilon_x = \frac{\dfrac{1540.50(12)}{53.50} + 0.5(0.0) + 0.5(274.10 - 16.42)(\cot 20.65^\circ) - 42(0.153)202.5}{2\left[4570.38(473) + 28000(0.0) + 28500(42)(0.153)\right]}$$

ε_x = –0.000131

$$\upsilon_u = \frac{V_u - \varphi V_p}{\varphi b_v d_v} = \frac{274.10 - 0.9(16.42)}{0.9(8)(53.50)} = 0.673 \text{ ksi} \qquad \text{[LRFD Eq. 5.8.3.4.2-1]}$$

υ_u/f'_c = 0.673/5.683 = 0.118

υ_u/f'_c	ε_x x 1000		
	≤–0.200	–0.131	≤ –0.100
≤ 0.100	18.100		20.400
	3.790		3.380
0.118	19.396	20.83	21.480
	3.351	3.18	3.099
≤ 0.125	19.90		21.900
	3.180		2.990

$\theta = 20.83^0 \approx 20.65^0$ (assumed)

Therefore no further iteration is needed.

$\beta = 3.18$

where β = a factor indicating the ability of diagonally cracked concrete to transmit tension.

A.2.11.2.3 Computation of Concrete Contribution

The nominal shear resisted by the concrete is:

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v \quad \text{[LRFD Eq. 5.8.3.3-3]}$$

$$V_c = 0.0316(3.18)\sqrt{5.683}(8)(53.50) = 102.53 \text{ kips}$$

A.2.11.3 Contribution of Reinforcement to Nominal Shear Resistance

A.2.11.3.1 Requirement for Reinforcement

Check if $V_u > 0.5\varnothing(V_c + V_p)$ [LRFD Eq. 5.8.2.4-1]

V_u = 274.10 kips > 0.5(0.9)(102.53 + 16.42) = 53.53 kips

Therefore, transverse shear reinforcement should be provided.

A.2.11.3.2 Required Area of Reinforcement

$$\frac{V_u}{\varphi} \leq V_n = (V_c + V_s + V_p) \quad \text{[LRFD Eq. 5.8.3.3-1]}$$

where

V_s = shear force carried by transverse reinforcement

$$= \frac{V_u}{\varphi} - V_c - V_p = \left(\frac{274.10}{0.9} - 102.53 - 16.42\right) = 185.61 \text{ kips}$$

$$V_s = \frac{A_v f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}{s} \quad \text{[LRFD Eq. 5.8.3.3-4]}$$

where

A_v = area of shear reinforcement within a distance s, in.^{2}

s = spacing of stirrups, in.

f_y = yield strength of shear reinforcement, ksi

α = angle of inclination of transverse reinforcement to longitudinal axis

= 90^0 for vertical stirrups

Therefore, area of shear reinforcement within a distance s is:

$A_v = (sV_s)/ f_y d_v(\cot\theta + \cot\alpha)\sin\alpha$

$= s(185.61)/(60)(53.50)(\cot 20.65^0 + \cot 90^0)\sin 90^0 = 0.0218(s)$

If s = 12 in., required $A_v = 0.262$ in^2 / ft

A.2.11.3.4 Determine spacing of reinforcement

Check for maximum spacing of transverse reinforcement [LRFD Art.. 5.8.2.7]

check if $v_u < 0.125f'_c$ [LRFD Eq. 5.8.2.7-1]

or if $v_u \geq 0.125f'_c$ [LRFD Eq. 5.8.2.7-2]

$0.125f'_c = 0.125(5.683) = 0.71$ ksi

$v_u = 0.673$ ksi

Since $v_u < 0.125f'_c$ therefore $s \leq 24$ in. [LRFD Eq. 5.8.2.7-2]

$s \leq 0.8d_v = 0.8(53.50) = 42.80$ in.

Therefore maximum s = 24.0 in. > s provided O.K.

Use #4 bar double legs at 12 in. c/c, $A_v = 2(0.20) = 0.40$ in^2/ft > 0.262 in^2/ft

$$V_s = \frac{0.4(60)(53.50)(\cot 20.65^0)}{12} = 283.9 \text{ kips}$$

A.2.11.3.4 Minimum Reinforcement requirement

The area of transverse reinforcement should not be less than: [LRFD Art. 5.8.2.5]

$$0.0316\sqrt{f'_c}\,\frac{b_v s}{f_y}$$ [LRFD Eq. 5.8.2.5-1]

$$= 0.0316\sqrt{5.683}\,\frac{(8)(12)}{60} = 0.12 < A_v \text{ provided} \qquad \text{O.K.}$$

A.2.11.4 Maximum Nominal Shear Resistance

In order to assure that the concrete in the web of the beam will not crush prior to yield of the transverse reinforcement, the LRFD Specifications give an upper limit for V_n as follows:

$V_n = 0.25f'_c b_v d_v + V_p$ [LRFD Eq. 5.8.3.3-2]

Comparing above equation with LRFD Eq. 5.8.3.3-1

$V_c + V_s \leq 0.25f'_c b_v d_v$

$102.53 + 283.9 = 386.43 \text{ kips} \leq 0.25(5.683)(8)(53.50) = 608.08$ kips O.K.

This is a sample calculation for determining transverse reinforcement requirement at critical section and this procedure can be followed to find the transverse reinforcement requirement at increments along the length of the beam.

A.2.12
INTERFACE SHEAR TRANSFER

A.2.12.1
Factored Horizontal Shear

[LRFD Art. 5.8.4]

At the strength limit state, the horizontal shear at a section can be calculated as follows

$$V_h = \frac{V_u}{d_v}$$ [LRFD Eq. C5.8.4.1-1]

where

V_h = horizontal shear per unit length of the beam, kips

V_u = factored shear force at specified section due to superimposed loads, kips

d_v = the distance between resultants of tensile and compressive forces (d_e-a/2), in

The LRFD Specifications do not identify the location of the critical section. For convenience, it will be assumed here to be the same location as the critical section for vertical shear, at point 0.057L

Using load combination Strength I:

V_u = 1.25(5.29) +1.50(6.06) + 1.75(66.81 + 25.15) = 176.63 kips

d_v = 53.50 in

Therefore applied factored horizontal shear is:

$$V_h = \frac{176.63}{53.50} = 3.30 \text{ kips/in.}$$

A.2.12.2
Required Nominal Resistance

Required $V_n = V_h / \varphi$ = 3.30/0.9 = 3.67 kip/in

A.2.12.3
Required Interface Shear Reinforcement

The nominal shear resistance of the interface surface is:

$V_n = cA_{cv} + \mu\ [A_{vf}f_y + P_c]$ [LRFD Eq. 5.8.4.1-1]

where

c = cohesion factor [LRFD Art. 5.8.4.2]

μ = friction factor [LRFD Art. 5.8.4.2]

A_{cv} = area of concrete engaged in shear transfer, in^2.

A_{vf} = area of shear reinforcement crossing the shear plane, in^2.

P_c = permanent net compressive force normal to the shear plane, kips

f_y = shear reinforcement yield strength, ksi

For concrete placed against clean, hardened concrete and free of laitance, but not an intentionally roughened surface: [LRFD Art. 5.8.4.2]

$c = 0.075$ ksi

$\mu = 0.6\lambda$, where $\lambda = 1.0$ for normal weight concrete, and therefore,

$\mu = 0.6$

The actual contact width, b_v, between the slab and the beam is 20 in.

$A_{cv} = (20 \text{ in.})(1 \text{ in}) = 20 \text{ in}^2$

The LRFD Eq. 5.8.4.1-1 can be solved for A_{vf} as follows:

$3.67 = (0.075)(20) + 0.6(A_{vf}(60) + 0)$

Solving for $A_{vf} = 0.06 \text{ in}^2/\text{in}$ or $0.72 \text{ in}^2 / \text{ft}$

2 - #4 double-leg bar per ft are provided.

Area of steel provided = 2 (0.40) = 0.80 in^2 / ft

Provide 2 legged #4 bars at 6 in. c/c

The web reinforcement shall be provided at 6 in. c/c which can be extended into the cast in place slab to account for the interface shear requirement.

A.2.12.3.1 Minimum Interface shear reinforcement

Minimum $A_{vf} \geq (0.05b_v)/f_y$ [LRFD Eq. 5.8.4.1-4]

where b_v = width of the interface

$A_{vf} = 0.80 \text{ in.}^2/\text{ft.} > [0.05(20)/60](12 \text{ in./ft}) = 0.2 \text{ in.}^2/\text{ft.}$ O.K.

A.2.12.4 Minimum Interface shear reinforcement

$$V_n \text{ provided} = 0.075(20) + 0.6\left(\frac{0.80}{12}(60) + 0\right) = 3.9 \text{ kips/in.}$$

$0.2f'_cA_{cv} = 0.2(4.0)(20) = 16$ kips/n.

$0.8A_{cv} = 0.8(20) = 16$ kips/in.

Since provided $V_n \leq 0.2\, f'_cA_{cv}$ O.K. [LRFD Eq. 5.8.4.1-2]

$\leq 0.8A_{cv}$ O.K. [LRFD Eq. 5.8.4.1-3]

A.2.13 MINIMUM LONGITUDINAL REINFORCEMENT REQUIREMENT

[LRFD Art. 5.8.3.5]

Longitudinal reinforcement should be proportioned so that at each section the following equation is satisfied

$$A_sf_y + A_{ps}f_{ps} \geq \frac{M_u}{d_v\varphi} + 0.5\frac{N_u}{\varphi} + \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta \quad \text{[LRFD Eq. 5.8.3.5-1]}$$

where

A_s = area of non prestressed tension reinforcement, in.^2

f_y = specified minimum yield strength of reinforcing bars, ksi

A_{ps} = area of prestressing steel at the tension side of the section, in.^{2}

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required, ksi

M_u = factored moment at the section corresponding to the factored shear force, kip-ft.

N_u = applied factored axial force, kips

V_u = factored shear force at the section, kips

V_s = shear resistance provided by shear reinforcement, kips

V_p = component in the direction of the applied shear of the effective prestressing force, kips

d_v = effective shear depth, in.

$\varnothing$ = resistance factor as appropriate for moment, shear and axial resistance. Therefore different $\varnothing$ factors will be used for terms in Eq. 5.8.3.5-1, depending on the type of action being considered [LRFD Art. 5.5.4.2]

θ = angle of inclination of diagonal compressive stresses.

A.2.13.1 Required Reinforcement at Face of Bearing

[LRFD Art. 5.8.3.5]

Width of bearing = 6.5 in.

Distance of section = 6.5/2 = 3.25 in. = 0.271 ft.

Shear forces and bending moment are calculated at this section

V_u = 1.25(44.35 + 43.22 + 5.94) + 1.50(6.81) + 1.75(71.05 + 28.14)

= 300.69 kips.

M_u = 1.25(12.04 + 11.73 + 1.61) + 1.50(1.85) + 1.75(15.11 + 6.00)

= 71.44 Kip-ft.

$$\frac{M_u}{d_v\varphi} + 0.5\frac{N_u}{\varphi} + \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta$$

$$= \frac{71.44(12)}{53.50(0.9)} + 0 + \left(\frac{300.69}{0.9} - 0.5(283.9) - 16.42\right)\cot 20.65^0 = 484.09 \text{ kips}$$

The crack plane crosses the centroid of the 42 straight strands at a distance of 6 + 5.33 cot 20.65^0 = 20.14 in. from the end of the beam. Since the transfer length is 30 in. the available prestress from 42 straight strands is a fraction of the effective prestress, f_{pe}, in these strands. The 10 harped strands do not contribute the tensile capacity since they are not on the flexural tension side of the member.

Therefore available prestress force is:

$$A_sf_y + A_{ps}f_{ps} = 0 + 42(0.153)\left(149.18\frac{20.33}{30}\right) = 649.63 \text{ Kips}$$

$$A_sf_y+A_{ps}f_{ps}=649.63 \text{ kips} > \frac{M_u}{d_v\varphi} + 0.5\frac{N_u}{\varphi} + \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta = 484.09 \text{ kips}$$

Therefore additional longitudinal reinforcement is not required.

A.2.14 PRETENSIONED ANCHORAGE ZONE

[LRFD Art. 5.10.10]

A.2.14.1 Minimum Vertical Reinforcement

[LRFD Art. 5.10.10.1]

Design of the anchorage zone reinforcement is computed using the force in the strands just prior to transfer:

Force in the strands at transfer = F_{pi} = 52 (0.153)(202.5) = 1611.09 kips

The bursting resistance, P_r, should not be less than 4% of F_{pi}

[LRFD Arts. 5.10.10.1 and C3.4.3]

$P_r = f_sA_s \geq 0.04F_{pi} = 0.04(1611.09) = 64.44$ kips

where

A_s = total area of vertical reinforcement located within a distance of h/4 from the end of the beam, in ^{2}.

f_s = stress in steel not exceeding 20 ksi.

Solving for required area of steel A_s= 64.44/20 = 3.22 in^2

Atleast 3.22 in^2 of vertical transverse reinforcement should be provided within a distance of (h/4 = 62 / 4 = 15.5 in). from the end of the beam.

Use 6 - #5 double leg bars at 2.0 in. spacing starting at 2 in. from the end of the beam.

The provided A_s = 6(2)0.31 = 3.72 in^2 > 3.22 in^2 O.K.

A.2.14.2 Confinement Reinforcement

[LRFD Art. 5.10.10.2]

For a distance of 1.5d = 1.5(54) = 81 in. from the end of the beam, reinforcement is placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than #3 deformed bars with spacing not exceeding 6 in. The reinforcement should be of shape which will confine the strands.

A.2.15 DEFLECTION AND CAMBER

A.2.15.1 Maximum Camber calculations using Hyperbolic Functions Method

TxDOT's prestressed bridge design software, PSTRS 14 uses the Hyperbolic Functions Method proposed by Sinno Rauf, and Howard L Furr (1970) for the calculation of maximum camber. This design example illustrates the PSTRS 14 methodology for calculation of maximum camber.

Step1: Total Prestress after release

$$P = \frac{P_{si}}{\left(1 + pn + \dfrac{e_c^2 A_s n}{I}\right)} + \frac{M_D e_c A_s n}{I\left(1 + pn + \dfrac{e_c^2 A_s n}{I}\right)}$$

where

P_{si} = total prestressing force = 1461.28 Kips

I = moment of inertia of non-composite section = 260403 in.^{4}

e_c = eccentricity of pretensioning force at the midspan = 19.29 in.

M_D = Moment due to self weight of the beam at midspan = 1209.98 K-ft.

A_s = Area of strands = number of strands (area of each strand)

= 52(0.153) = 7.96 in.^{2}

p = As/A

where

A = Area of cross section of beam = 788.4 in.^{2}

p = 7.96/788.4 = 0.0101

E_c = modulus of elasticity of the beam concrete at release, ksi

$= 33(w_c)^{3/2}\sqrt{f'_c}$ [STD Eq. 9-8]

$= 33(150)^{1.5}\sqrt{5683.33}\ \dfrac{1}{1000} = 4570.38$ ksi

E_p = Modulus of elasticity of prestressing strands = 28500 ksi

$n = E_p/E_c$ = 28500/4570.38 = 6.236

$$\left(1 + pn + \frac{e_c^2 A_s n}{I}\right) = 1 + (0.0101)(6.236) + \frac{(19.29^2)(7.96)(6.236)}{260403} = 1.134$$

$$P = \frac{P_{si}}{\left(1 + pn + \dfrac{e_c^2 A_s n}{I}\right)} + \frac{M_D e_c A_s n}{I\left(1 + pn + \dfrac{e_c^2 A_s n}{I}\right)}$$

$$= \frac{1461.28}{1.134} + \frac{(1209.98)(12\text{ in./ft.})(19.29)(7.96)(6.236)}{260403(1.134)}$$

= 1288.61 + 47.08 = 1335.69 Kips

Concrete Stress at steel level immediately after transfer

$$f^s_{ci} = P\left(\frac{1}{A} + \frac{e_c^2}{I}\right) - f^s_c$$

where

f^s_c = Concrete stress at steel level due to dead loads

$$= \frac{M_D e_c}{I} = \frac{(1209.98)(12 \text{ in./ft.})(19.29)}{260403} = 1.0756 \text{ ksi}$$

$$f^s_{ci} = 1335.69\left(\frac{1}{788.4} + \frac{19.29^2}{260403}\right) - 1.0756 = 2.527 \text{ ksi}$$

Step2: Ultimate time-dependent strain at steel level

$\varepsilon^s_{c1} = \varepsilon^\infty_{cr} f^s_{ci} + \varepsilon^\infty_{sh}$

where

ε^∞_{cr} = ultimate unit creep strain = 0.00034 in./in. (this value is prescribed by Sinno et. al. (1970))

ε^∞_{sh} = ultimate unit creep strain = 0.000175 in./in. (this value is prescribed by Sinno et. al. (1970))

ε^s_{c1} = 0.00034(2.527) + 0.000175 = 0.001034 in./in.

Step3: Adjustment of total strain in step 2

$$\varepsilon^s_{c2} = \varepsilon^s_{c1} - \varepsilon^s_{c1} E_p \frac{A_s}{E_c}\left(\frac{1}{A} + \frac{e_c^2}{I}\right)$$

$$= 0.001034 - 0.001034(28500)\frac{7.96}{4570.38}\left(\frac{1}{788.4} + \frac{19.29^2}{260403}\right) = 0.000896 \text{ in./in.}$$

Step4: Change in concrete stress at steel level

$$\Delta f^s_c = \varepsilon^s_{c2} E_p A_s\left(\frac{1}{A} + \frac{e_c^2}{I}\right) = 0.000896(28500)(7.96)\left(\frac{1}{788.4} + \frac{19.29^2}{260403}\right)$$

Δf^s_c = 0.548 ksi

Step5: Correction of the total strain from step2

$$\varepsilon^{s}_{c4} = \varepsilon^{\infty}_{cr}\left(f^{s}_{ci} - \frac{\Delta f^{s}_{c}}{2}\right) + \varepsilon^{\infty}_{sh}$$

$$\varepsilon^{s}_{c4} = 0.00034\left(2.527 - \frac{0.548}{2}\right) + 0.000175 = 0.000941 \text{ in./in.}$$

Step6: Adjustment in total strain from step 5

$$\varepsilon^{s}_{c5} = \varepsilon^{s}_{c4} - \varepsilon^{s}_{c4}\, E_{s} \frac{A_{s}}{E_{c}}\left(\frac{1}{A} + \frac{e_{c}^{2}}{I}\right)$$

$$= 0.000941 - 0.000941(28500)\frac{7.96}{4570.38}\left(\frac{1}{788.4} + \frac{19.29^{2}}{260403}\right) = 0.000815 \text{ in./in}$$

Step 7: Change in concrete stress at steel level

$$\Delta f^{s}_{c1} = \varepsilon^{s}_{c5}\, E_{p}\, A_{s}\left(\frac{1}{A} + \frac{e_{c}^{2}}{I}\right) = 0.000815(28500)(7.96)\left(\frac{1}{788.4} + \frac{19.29^{2}}{260403}\right)$$

$\Delta f^{s}_{c1} = 0.499$ ksi

Step 8: Correction of the total strain from step 5

$$\varepsilon^{s}_{c6} = \varepsilon^{\infty}_{cr}\left(f^{s}_{ci} - \frac{\Delta f^{s}_{c1}}{2}\right) + \varepsilon^{\infty}_{sh}$$

$$\varepsilon^{s}_{c6} = 0.00034\left(2.527 - \frac{0.499}{2}\right) + 0.000175 = 0.000949 \text{ in./in.}$$

Step9: Adjustment in total strain from step 8

$$\varepsilon^{s}_{c7} = \varepsilon^{s}_{c6} - \varepsilon^{s}_{c6}\, E_{p} \frac{A_{s}}{E_{c}}\left(\frac{1}{A} + \frac{e_{c}^{2}}{I}\right)$$

$$= 0.000949 - 0.000949(28500)\frac{7.96}{4570.38}\left(\frac{1}{788.4} + \frac{19.29^{2}}{260403}\right) = 0.000822 \text{ in./in}$$

Step 10: Computation of initial prestress loss

$$PL_{i} = \frac{\text{Psi - P}}{\text{Psi}} = \frac{1461.28 - 1335.69}{1461.28} = 0.0859$$

Step 11: Computation of Final Prestress loss

$$PL^{\infty} = \frac{\varepsilon^{s}_{c7} E_p A_s}{P_{si}} = \frac{0.000822(28500)(7.96)}{1461.28} = 0.128$$

Total Prestress loss

$PL = PL_i + PL^{\infty} = 100(0.0859 + 0.128) = 21.39\%$

Step 12: Initial deflection due to dead load

$$C_{DL} = \frac{5wL^4}{384E_cI}$$

where

w = weight of beam = 0.821 kips/ft.

L = span length = 108.583 ft.

$$C_{DL} = \frac{5\left(\frac{0.821}{12\text{ in./ft.}}\right)[(108.583)(12\text{ in./ft.})]^4}{384(4570.38)(260403)} = 2.158\text{ in.}$$

Step 13: Initial Camber due to prestress

C_{pi}=[0.5(P)(e_e)(0.5L)^2 + 0.5(P)($e_c - e_e$)(0.67)(HD)^2

+0.5P ($e_c - e_e$)(HDdis)(0.5L + HD)]/(E_c)(I)

where

HD = Hold down distance from support = 48.87 ft. = 586.44 in.

HDdis = Hold down distance from center

= 0.5(108.583) – 48.87 = 5.422 ft. = 65.06 in.

e_e = Eccentricity at support = 11.21 in.

L = span length = 108.583 ft. = 1303 in.

C_{pi} = {0.5(1335.69)(11.21)[(0.5)(1303)]^2 + 0.5(1335.69)(19.29 – 11.21)(0.67)(586.44)^2

+ 0.5(1335.69)(19.29 – 11.21)(65.06)(0.5(1303) + 586.44)}/[(4570.38)(260403)]

C_{pi} = [3.178*10^9 + 1.243*10^9 + 0.4346*10^9]/1.19*10^9 = 4.08 in.

Step 14: Initial Camber

$C_i = C_{pi} - C_{DL}$ = 4.080 – 2.158 = 1.922 in.

Step 15: Ultimate Time Dependent Camber

Ultimate strain $\varepsilon^{s}_{e} = f^{s}_{ci}/E_c$ = 2.527/4570.37 = 0.000553 in./in.

$$\text{Ultimate camber } C_t = C_i(1 - PL^{\infty})\frac{\varepsilon^{\infty}{}_{cr}\left(f^{s}{}_{ci} - \dfrac{\Delta f^{s}{}_{c1}}{2}\right) + \varepsilon^{s}{}_{e}}{\varepsilon^{s}{}_{e}}$$

$$= 1.922(1 - 0.128)\frac{0.00034\left(2.527 - \dfrac{0.499}{2}\right) + 0.000553}{0.000553}$$

C_t = 4.023 in. = 0.335 ft. ↑

A.2.15.2 Deflection due to slab weight

Deflection at midspan due to slab weight

$$\Delta_{slab1} = \frac{5w_sL^4}{384E_cI}$$

where

w_s = slab weight = 0.8 Kips/ft.

E_c = modulus of elasticity of beam concrete at service

$$= (150)^{1.5}(33)\sqrt{5683.33}\ \frac{1}{1000} = 4570.38 \text{ ksi}$$

I = moment of inertia of non-composite section = 260403 in.^{4}

L = span length = 108.583 ft.

$$\Delta_{slab1} = \frac{5(0.8/12)[(108.583)(12)]^4}{384(4570.38)(260403)} = 2.10 \text{ in.} = 0.175 \text{ ft.}\downarrow$$

Deflection at quarter span due to slab weight

$$\Delta_{slab2} = \frac{57w_sL^4}{6144E_cI}$$

$$\Delta_{slab2} = \frac{57(0.8/12)[(108.583)(12)]^4}{6144(4570.38)(260403)} = 1.498 \text{ in.} = 0.1248 \text{ ft.}\downarrow$$

A.2.15.3 Deflection due to Super Imposed loads

Deflection due to barrier weight at midspan

$$\Delta_{barr1} = \frac{5w_{barr}L^4}{384E_cI_c}$$

where

w_{barr} = barrier weight = 0.11 Kips/ft.

I_c = moment of inertia of composite section = 694599.5 in.^{4}

$$\Delta_{barr1} = \frac{5(0.11/12)[(108.583)(12)]^4}{384(4570.38)(694599.5)} = 0.108 \text{ in.} = 0.009 \text{ ft.}\downarrow$$

Deflection at quarter span due to barrier weight

$$\Delta_{barr2} = \frac{57 w_{barr} L^4}{6144 E_c I_c}$$

$$\Delta_{barr2} = \frac{57(0.11/12)[(108.583)(12)]^4}{6144(4570.38)(694599.5)} = 0.0772 \text{ in.} = 0.0064 \text{ ft.}$$

Deflection due to wearing surface weight at midspan

$$\Delta_{ws1} = \frac{5 w_{ws} L^4}{384 E_c I_c}$$

where

w_{ws} = wearing surface weight = 0.1259 Kips/ft.

I_c = moment of inertia of composite section = 694599.5 in.^{4}

$$\Delta_{ws1} = \frac{5(0.1259/12)[(108.583)(12)]^4}{384(4570.38)(694599.5)} = 0.124 \text{ in.} = 0.01 \text{ ft.}\downarrow$$

Deflection at quarter span due to barrier weight

$$\Delta_{ws2} = \frac{57 w_{ws} L^4}{6144 E_c I_c}$$

$$\Delta_{ws2} = \frac{57(0.1259/12)[(108.583)(12)]^4}{6144(4570.38)(694599.5)} = 0.0884 \text{ in.} = 0.0074 \text{ ft.}\downarrow$$

A.1.16 COMPARISON OF RESULTS OBTAINED FROM STANDARD DETAILED DESIGN AND PSTRS 14 WITH THAT OF THE LRFD DETAILED DESIGN

The prestressed beam design program, PSTRS14 is used by TxDOT to design the bridges. PSTRS14 program was run with same parameters as used in this detailed design and the results are compared in table A.1.16.1. Also the results from detailed design using AASHTO Standard Specifications are compared.

Table A.1.16.1 Comparison of results obtained from PSTRS with that of the detailed design

Parameter	LRFD Detailed Design Results	PSTRS 14 Results	Percent Difference	Standard Detailed Design Results	Percent Difference
Live Load Distribution factor Moment	0.639	0.727	***-13.77 %***	0.727	***-13.77 %***
Shear	0.814	0.727	***10.69 %***	0.727	***10.69 %***
Initial Prestress loss	9.30 %	8.93%	3.98 %	8.94%	3.87 %
Final Prestress Loss	26.33 %	25.23%	4.18 %	25.25%	4.10 %
Beam Stresses at release					
At beam end: at top fiber	13 psi	35 psi	***-169.23 %***	35 psi	***-169.23 %***
at bottom fiber	3410 psi	3274 psi	3.99 %	3274 psi	3.99 %
At transfer length section: at top fiber	94 psi	Not Calculated	-	103 psi	-9.57 %
At bottom fiber	3342 psi	Not calculated	-	3216 psi	3.77 %
At Hold Down: at top fiber	334 psi	319 psi	4.49 %	354 psi	-5.99 %
At bottom fiber	3138 psi	3034 psi	3.31 %	3005 psi	4.24 %
At Midspan: at top fiber	351 psi	335 psi	4.56 %	368 psi	-4.84 %
At bottom fiber	3124 psi	3020 psi	3.33 %	2991 psi	4.26 %
Beam Stresses at service loads					
At beam end: at top fiber	Not Calculated	29 psi	-	Not Calculated	-
At bottom fiber	Not Calculated	2688 psi	-	Not Calculated	-
At Midspan under total loads: at top fiber	2679 psi	2562 psi	4.37 %	2562 psi	4.37 %
At bottom fiber	-444 psi	-413 psi	6.98 %	-411 psi	7.43 %
Slab Top Stresses under total loads	855 psi	Not Caclulated	-	660 psi	22.81 %
Release Concrete Strength required	5683.33 psi	5457 psi	3.98 %	5457 psi	3.98 %
Final Concrete Strength required	5683.33 psi	5585 psi	1.73 %	5585 psi	1.73 %
Total No. of Strands	52	50	3.85 %	50	3.85 %
No. of Draped Strands	10	10	0.00 %	10	0.00 %
Ultimate Moment Required	7012.47 K-ft.	6769 K-ft.	3.47 %	6767.27 K-ft.	3.50 %
Ultimate Moment Provided	9201.15 K-ft.	8805 K-ft	4.31 %	8936.56 K-ft.	2.88 %
Shear Reinforcement	#4 bars at 6 in. c/c	#3 bars at 12 in. c/c		#3 bars at 12 in. c/c	
Area of shear Reinforcement	0.40 in.^{2}/ft.	0.11 in.^{2}/ft.	***72.50 %***	0.11 in.^{2}/ft.	***72.50 %***

Maximum Camber	0.335 ft.	0.318 ft.	5.35 %	0.320 ft.	5.33 %
Deflections					
Slab Weight: At midspan	-0.175 ft.	-0.1767 ft.	-0.97 %	-0.177 ft.	-1.14 %
At Quarter span	-0.1248 ft.	-0.1259 ft.	-0.88 %	-0.1259 ft.	-0.88 %
Barrier Weight: At midspan	-0.0090 ft.	-0.0091 ft.	-1.11 %	-0.0091 ft.	-1.11 %
At quarter span	-0.0064 ft.	-0.0065 ft.	-1.56 %	-0.0065 ft.	-1.56 %
Wearing surface weight: At midspan	-0.01 ft.	-0.0104 ft.	-4.00 %	-0.01 ft.	0.00 %
At quarter span	-0.0074 ft.	-0.0074 ft.	0.00 %	-0.0074 ft.	0.00 %